WEAKLY NONLINEAR THEORY FOR RAYLEIGH-TAYLOR INSTABILITY OF ROTATING FLUIDS

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Summary Weakly nonlinear theory of Rayleigh-Taylor instability (RTI) is developed for a rotating system, where the rotating axis is normal to the acceleration of the interface between two uniform and inviscid fluids. It is shown that RTI can be inhibited completely by rotation at arbitrary Atwood numbers. Increasing rotating velocity will strengthen the high-order suppression effect on the basic mode disturbance. The suppression mechanism is explained in terms of the restoring effect of Coriolis force induced by the rotation.

INTRODUCTION

The instability of an interface, which separates two fluids and is accelerated into the heavier fluid, is known as the Rayleigh-Taylor instability (RTI) in honor of Lord Rayleigh, who studied the unstable mode under gravitational field, and Sir G.I. Taylor, who considered the linear stability of unstable surface waves under acceleration. RTI is of interest in a wide variety of fields, such as deep convection in oceans, inertial-confinement fusion, supernovae explosions and plasma physics. There are three stages in the RTI development. Interface disturbances in the linear stage grow exponentially with a growth rate:

$$\gamma = \sqrt{gkA},\ A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}, \tag{1}$$

where g is the interface acceleration, k is the wavenumber in the plane normal to g and A is the Atwood number. ρ_1 and ρ_2 are the densities of the denser and lighter fluids, respectively. Along with the growth of perturbations, nonlinear interface is twisted into bubbles of lighter fluid and spikes of the heavier one penetrating into the lighter fluid. At the last stage, the strong shear near the interface may cause vortices due to Kelvin-Helmholtz instability and then extensive interfacial mixing ensues.

Since RTI may dramatically reduce the performance of inertial confinement fusion at both the initial implosion stage and later deceleration stage by RT mixing, RTI should be suppressed as completely as possible. In order to study the nonlinear behavior of bubbles and spikes, weakly nonlinear theory was developed for two-dimensional, three-dimensional geometries [1] and arbitrary Atwood numbers [2] of potential flows. Hide [3] carried out normal mode analysis for exponentially stratified incompressible and viscous fluids. He found that rotation tended to inhibit RTI through viscous effect. By numerical simulations, Carnevale [4] demonstrated recently that rotation with viscosity and diffusion, at least in the Boussinesq approximation or for low Atwood number flow, could suppress RTI and RT mixing. In their studies the rotating axis is parallel to the gravity and the RTI suppression is explained by analyzing the vorticity field in the mixing zone. For inviscid and irrotational uniform fluids, Chandrasekhar [5] has shown that a uniform rotation about the direction of the interface acceleration decreases the growth rate of normal mode disturbances. Following this pioneer work a series of linear RTI analysis have been carried out to study the effect of rotation combined with viscosity, general rotation, and surface tension.

Previous stability analyses of rotating RTI are focused on the linear stage, and hence the nonlinear effect of rotation on the bubble/spike formation, especially when the rotating vector is normal to the acceleration, has not been studied theoretically so far. This is the motivation of the present paper.

BASIC MODEL AND WEAKLY NONLINEAR THEORY

We consider two inviscid fluids which are immiscible and rotate around the axis z with a constant angular velocity Ω. The circular interface at $r = R$ is accelerated to the inner fluid due to pressure gradient, and the densities of inner and outer fluids near the interface are assumed to be constant values ρ_1 and ρ_2, respectively. The disturbed flow can be described in a Cartesian coordinates fixed at the undisturbed interface when the disturbance amplitude is much smaller than R. x and y points to the azimuthal and radial directions, respectively. η is the displacement of the disturbed interface and the acceleration g is in the same direction as y.

The two-dimensional potential flow of fluids is described by hydrodynamical potentials and stream functions. To the first order approximation, it is easy to solve the frequency of unstable normal mode as $\omega = \gamma - i\Omega A$, where $\gamma = \sqrt{gkA - A^2\Omega^2}$ is the linear growth rate, k is the wave number in the x direction and the Atwood number is $A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$.

For the case of a basic-mode initial disturbance of η in the form $\epsilon \cos(kx)$, the corresponding unstable solution up to third order is obtained following the method proposed by Jacobs and Catton [1],

$$\begin{aligned} \eta = \quad & \epsilon \cos(kx - A\Omega t)e^{\gamma t} + \epsilon^2 C_2 \cos(2kx - 2A\Omega t)e^{2\gamma t} \\ + \quad & \epsilon^3[C_{31}^c \cos(kx - A\Omega t) + C_{31}^s \sin(kx - A\Omega t) + C_3 \cos(3kx - 3A\Omega t)]e^{3\gamma t}, \end{aligned} \tag{2}$$

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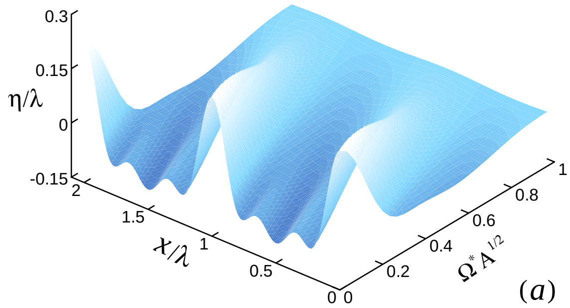

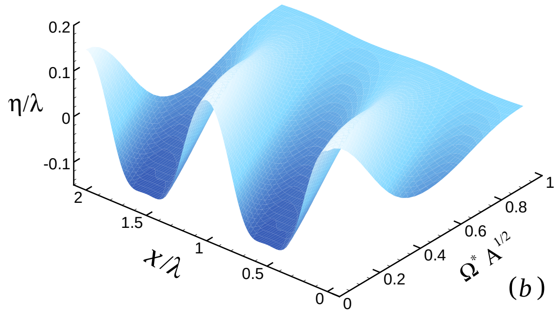

Figure 1. Interface displacements η at $\gamma_0 t = 5$ and $\epsilon k = 0.001$ for (a)A=1 and (b)A=0.4, respectively. $\gamma_0 = \sqrt{gkA}$, $\Omega^* = \Omega/\sqrt{gk}$ and $\lambda = 2\pi/k$.

where $C_2 = \frac{kA}{2}$, $C_{31}^c = -\frac{k^2}{16\gamma^2}(A^4\Omega^2 + A^2\Omega^2 + 3A^2\gamma^2 + \gamma^2)$, $C_{31}^s = \frac{k^2 A\Omega}{8\gamma}(A^2+1)$ and $C_3 = \frac{k^2}{8}(4A^2-1)$. When the angular velocity Ω decreases to zero the linear growth rate γ decays to the traditional value $\gamma_0 = \sqrt{gkA}$, and when $A = 1$ and $\Omega = 0$ the interface position η (equation 2) is just the solution given by Jacobs and Catton [1].

The uniform rotation brings several new features for the Rayleigh-Taylor instability. Firstly, according to the expression of the linear growth rate $\gamma = \sqrt{gkA - A^2\Omega^2}$, the rotation, whose direction is normal to the acceleration, suppresses RTI completely when the dimensionless angular velocity $\Omega^* = \Omega/\sqrt{gk} = 1/\sqrt{A}$. Secondly, since only the product gA appears in γ, the suppression effect on the linear RTI exists for both an acceleration stage ($g > 0$ and $A > 0$) and a deceleration stage ($g < 0$ and $A < 0$). Thirdly, the unstable modes are traveling waves. For example, the basic mode has a phase velocity of $A\Omega/k$.

According to equation (2), the nonlinear evolution of the basic mode up to the third order is expressed as

$$\eta_b \cos(kx - A\Omega t) = (\eta_L + \eta_L^3 C_{31}^c)\cos(kx - A\Omega t), \quad \eta_L = \epsilon e^{t\gamma}. \tag{3}$$

Since $C_{31}^c < 0$ the third-order modulation decreases the growth of the basic mode, and hence delays the growth of bubbles and spikes formed at the interface. In addition, larger rotation velocity and Atwood number will cause stronger suppression of the nonlinear evolution of RTI. The spikes and bubbles, where higher order harmonics are generated, can be annihilated by rotation as shown in figure 1(a) and (b). Since the linear growth rate γ is decreased by rotation, the growth rates of harmonics (e.g. 2γ, 3γ, etc.) become smaller as well, hence the harmonics are also suppressed by the rotation, though the coefficients of harmonic terms (e.g. C_2 and C_3 of equation 2) are independent of the rotation.

The suppression mechanism may be explained as follows. To the first order approximation, the x-direction disturbing velocity at the interface can be expressed as $u_1 \simeq -u_2 \simeq \epsilon e^{\gamma t} cos(kx - A\Omega t + \theta)$, where θ is the rotation-induced phase shift between u and the interface perturbation η. $cos\theta = A\Omega/\gamma_0 = \sqrt{A}\Omega^*$. $\theta = \pi/2$ for classical RTI case without rotation. Since u_1 and u_2 share the same absolute value but point to contradict directions at the interface, the direction of the effective normal Coriolis force is the same as that of the heavier side. When Ω^* increases to $1/\sqrt{A}$ or $\theta = 0$, u_1 is in the same phase as the interface displacement, and the effective normal Coriolis force becomes a restoring force every where on the interface. It is easy to check that when $\Omega < 0$ the suppression mechanism of the Coriolis force works as well.

CONCLUSIONS

In this paper, the first weakly nonlinear analysis is carried out for Rayleigh-Taylor instability of rotating fluids. It is shown that RTI can be suppressed completely by Coriolis force for arbitrary Atwood numbers. The new features of interface disturbances brought by the uniform rotation are discussed as well.

References

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