VORTEX-WAVE INTERACTION THEORY AND SELF-SUSTAINED PROCESSES IN SHEAR FLOWS

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Summary Recently Hall and Sherwin (2010) and Hall (2011) have shown that the vortex-wave interaction theory of Hall and Smith (1991) is the high Reynolds number counterpart of self-sustained processes calculated numerically at finite Reynolds numbers. The agreement of the theory with numerical calculations is excellent even at Reynolds numbers of size 1000 for Couette flow so the theory provides viable methods to understand high Reynolds number states not accessible computationally. Here theory is developed for internal and external shear flows and the effect of rough walls and flow oscillations on the development of equilibrium states are considered. Wall roughness is shown to be crucial in the selection process when multiple equilibrium states are possible. The approach reveals a mechanism for the random generation of localized patches of vortex-wave interactions in shear flows and is consistent with experimental observations.

VORTEX-WAVE INTERACTIONS IN THE PRESENCE OF ROUGHNESS AND FLOW OSCILLATIONS

Recently there has been immense interest in such self-sustained processes in shear flows. Sometimes these are referred to as exact coherent states and are typified by wave systems driving rolls which themselves drive streaks which are then neutrally stable to the wavefield. In fact this is exactly the same process described in a series of papers by Hall and Smith some 20-25 years ago; see for example Bennet et al (1991), Hall and Smith (1988, 1989, 1990, 1991). The latter authors described this kind of interaction by what they termed ' vortex-wave interaction theory'. The theory was based on asymptotic reductions of the Navier Stokes equations to much simpler systems describing the essential physics of the interaction. Some years later a number of papers investigated numerically what is essentially exactly the same kind of interaction, and, since the obvious connection of the latter work with vortex-wave interaction theory was not made, the structures found were referred to as self-sustained processes. Interestingly the mechanism developed independently by both groups was to a certain extent motivated by the work of Benney (1984). However Benney (1984) considered the situation when both the waves and vortex flows are inviscid. In the works of Hall and Smith on vortex-wave interaction theory the vortex flow is viscous; this enables analytical progress and the reduction of the interaction process down to relatively simple form. In particular when the wave is inviscid apart from a viscous critical layer the interaction between the wave and vortex flow is confined to the critical layer and the Reynolds stresses associated with the wave drive jumps in the vortex shear stresses across the critical layer.

Most of the numerical work has been in relation to channel flow simulations but there exists many closely related results for pipe flow; see for example Faisst and Eckhardt (2003), Wedin and Kerswell (2004), Kerswell and Tutty (2007), Viswanath (2007), and Schneider et al (2008). In channel flows Waleffe and co-workers, guided by the early ideas of Benney and co-workers, in a number of papers looked into the possibility that the Navier Stokes equations might support wave systems occurring as instabilities of streamwise vortex flows but sufficiently large to drive the vortex flows; see for example Waleffe (1995,1997,1998,2001,2003), Wang et al (2007). The approach used was to find fully nonlinear solutions of the Navier Stokes equations using various fictitious forces to find an equilibrium state and then continue them when the forcing was switched off. Earlier Nagata (1990) had done related calculations with the nonlinear flow initially driven by wall curvature and then connected to a finite amplitude state as the curvature decreased to zero. Much more detail of these structures was revealed subsequently in for example Clever and Busse (1997). The vortex-wave interaction theory most closely related to the latter body of work is Hall and Smith (1991) who considered the interaction of streamwise vortices with either Tollmient-Schlichting waves or inviscid waves in boundary layer flows. This problem is somewhat more difficult than the channel flow case because boundary layer growth must be accounted for but the interactions equations found there for inviscid wave/streamwise vortices are quite generic and so apply to a whole range of flows.

The asymptotic description given by Hall and Smith (1991) was somewhat complicated but Hall and Sherwin (2010) gave a relatively trivial derivation of the interaction equations based on what is essentially steady streaming theory. In fact Hall and Sherwin (2010) used the asymptotic framework derived by Hall and Smith (1991) in the context of vortex-wave interaction theory to give a high Reynolds number description of the lower branch self-sustained processes found by for example Waleffe. It was conclusively shown that the so-called self-sustained process description of interacting rolls, streaks and waves in channel flows is merely a finite Reynolds number version of the vortex-wave interaction theory applied to Rayleigh waves and longitudinal vortices. Here we will formulate vortex-wave interaction theory for natural convection flows between walls maintained at different temperatures and moving with equal and opposite velocities. This will enable us firstly lay down a framework to capture the essential details of self-sustained processes in turbulent natural convection and secondly for us to explore the origin of the lower branch modes of the type discussed by Hall and Sherwin (2010). The description we give is very much in spirit with the Hall and Smith (1991).

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Real flows do not take place in perfectly smooth channels or over perfectly smooth surface so a natural question is to determine what is the effect of wall roughness and indeed unsteady flow oscillations on the possible equilibrium states. In the general situation the wave will be propagating so that if the wave is to be generated by a receptivity process then both the wave frequency and wavelength must be stimulated. Wall roughness can be handled using a small amplitude expansion near the wall and the flow oscillations produce Stokes layers near the wall which interact with the flow induced by wall roughness to produce waves. In addition the wall roughness can directly stimulate stationary rolls and streaks so it is necessary to investigate how flow imperfections select whether to stimulate the wave or streamwise vortex flow. The outcome is to modify the interaction equations in such a way that the forcing dictates which equilibrium state is selected. In addition the time dependent version of the amplitude equations allow for the random generation of patches of vortex/wave activity in shear flows. These patches of activity are similar to patterns observed experimentally. The approach allows for wall roughness which stimulates a band of downstream and spanwise wavelengths and flow oscillations including a spread of frequencies. As such the approach models the quite general response of a vortex-wave interaction/self-sustained process to flow imperfections and captures some observations found experimentally.

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