LINEAR MECHANISMS OF THE SENSITIVITY OF JETS TO EXTERNAL FORCING.

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Summary The *preferred mode* of a subsonic jet is investigated within a linear framework. The computation of the most efficient time-harmonic forcing, as a function of excitation frequency, gives results consistent with the experimental literature with regard to both the Strouhal number and the spatial shape of the most amplified perturbations.

INTRODUCTION

The investigation of instability dynamics in jet flows is motivated by efforts to reduce jet noise in commercial aircraft. Isothermal jets do not exhibit self-sustained coherent oscillations but are known to strongly amplify incoming perturbations. Crow & Champagne [1] showed experimentally that this amplification significantly varies with the excitation frequency, and that optimal amplification is found for a Strouhal number of about 0.3, based on the jet diameter. More recent experimental and numerical studies report values between 0.25 and 0.5, depending on operating conditions.

Analysis of the flow dynamics in terms of temporal eigenmodes ("global modes") [7, 2] turned out to be computationally expensive and not well-suited to describe advection-dominated flows. The present study aims to demonstrate that a more suitable formalism for the study of the preferred mode in jets is that of optimal forcing [4, 6].

FLOW CONFIGURATION

A subsonic jet issuing from a cylindrical nozzle is considered. It is described by the compressible Navier–Stokes equations formulated in cylindrical coordinates (r, x, θ). The equations are made dimensionless using the jet radius R^*, the jet centerline velocity U_0^*, the density ρ_∞^* and the temperature T_∞^* in the ambient fluid at rest. The fluid is assumed to be a perfect Newtonian gas; furthermore, its adiabatic index $\gamma = 1.4$, thermal conductivity κ^*, specific heat at constant pressure C_p^* and viscosity μ^* are assumed to be constant. With this normalization, natural choices for the Reynolds, Mach and Prandtl numbers are

$$\mathrm{Re} = \frac{U_0^* R^* \rho_\infty^*}{\mu^*}, \qquad \mathrm{Ma} = \frac{U_0^*}{c_\infty^*}, \qquad \mathrm{Pr} = \frac{\mu^* C_P^*}{\kappa^*},$$

where $c_\infty^* = \sqrt{\gamma r^* T_\infty^*}$ denotes the ambient speed of sound.

The reference base flow used for linear stability studies is inspired by a model from [5] which reproduces experimentally measured mean flows in turbulent jets. It has been modified to take into account the presence of an infinitely thin adiabatic jet pipe, as described in [2]. In the present study, the flow parameters are $\mathrm{Re} = 10^3$, $\mathrm{Ma} = 0.75$ and $\mathrm{Pr} = 1$. The shear layer thickness is characterized by a momentum thickness Θ^* such that $R^*/\Theta^* = 23$ at the nozzle exit.

The Navier–Stokes equations are discretized in a matrix-free framework using high-resolution finite-difference schemes on a rectilinear mesh containing 1024×384 points. The invariance of the base flow in the azimuthal direction allows Fourier modes to be treated independently. This choice of discretization, together with appropriate boundary conditions allows an accurate treatment of acoustic radiation. See reference [2] for more details.

OPTIMAL HARMONIC FORCING

The optimization process is conveniently described within the discrete framework. The finite difference-discretization of the linearized Navier–Stokes equations leads to a linear ordinary equation of the form

$$\dot{\boldsymbol{q}} = L\boldsymbol{q} + B\boldsymbol{f} \tag{1}$$

In the above equation, $\boldsymbol{q}$ is the state vector, containing the values of the density, momentum and energy perturbations at each discretization point. The symbol L is a linear operator corresponding to the discretization of the equations, and $\boldsymbol{f}$ and B respectively denote the external forcing and its action on the equations. In the present study, the operator B imposes a localization of the forcing in the interior of the jet pipe. If the forcing is time harmonic, $\boldsymbol{f}(t) = \widetilde{\boldsymbol{f}} \exp(-\mathrm{i}\omega t)$ with $\omega \in \mathbb{R}$, then the time harmonic response is $\boldsymbol{q}(t) = \widetilde{\boldsymbol{q}} \exp(-\mathrm{i}\omega t)$ with $\widetilde{\boldsymbol{q}} = -(L + \mathrm{i}\omega \mathrm{Id})^{-1} B \widetilde{\boldsymbol{f}}$, where Id is the identity operator. Let the forcing and response intensities be respectively measured by $\|\widetilde{\boldsymbol{f}}\|^2 = \widetilde{\boldsymbol{f}}^\dagger Q_f \widetilde{\boldsymbol{f}}$ and $\|\widetilde{\boldsymbol{q}}\|^2 = \widetilde{\boldsymbol{q}}^\dagger Q_q \widetilde{\boldsymbol{q}}$, where Q_f is Hermitian definite. The optimal forcing problem consists in finding

$$\widetilde{\boldsymbol{f}}_{opt}(\omega) = \arg\max_{\widetilde{\boldsymbol{f}}} \frac{\|\widetilde{\boldsymbol{q}}\|^2}{\|\widetilde{\boldsymbol{f}}\|^2} = \arg\max_{\widetilde{\boldsymbol{f}}} \frac{\widetilde{\boldsymbol{f}}^\dagger B^\dagger (L^\dagger - \mathrm{i}\omega \mathrm{Id})^{-1} Q_q (L + \mathrm{i}\omega \mathrm{Id})^{-1} B \widetilde{\boldsymbol{f}}}{\widetilde{\boldsymbol{f}}^\dagger Q_f \widetilde{\boldsymbol{f}}}. \tag{2}$$

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Let $Q_f = M^\dagger M$ be the Cholesky factorization of Q_f. Equation (2) shows that the optimal forcing is given by $\widetilde{\boldsymbol{f}}_{opt}(\omega) = M^{-1}\boldsymbol{y}$, where $\boldsymbol{y}$ is the leading eigenvector of the Hermitian operator $M^{-1\dagger} B^\dagger (L^\dagger - \mathrm{i}\omega \mathrm{Id})^{-1} Q_q (L + \mathrm{i}\omega\mathrm{Id})^{-1} B M^{-1}$. In practice this problem is solved using the Lanczos solver implemented in SLEPc [8]. For standard norms, such as the compressible energy norm [3] that is used in this study, the operator M^{-1} is given analytically. In this case, the entire computational cost of the eigenvalue problem solution arises from the application of $(L + \mathrm{i}\omega\mathrm{Id})^{-1}$ and $(L^\dagger - \mathrm{i}\omega\mathrm{Id})^{-1}$. These could be handled using a direct or iterative sparse solver [4]. In order to keep the computational cost as low as possible, and to keep a matrix-free framework, $(L + \mathrm{i}\omega\mathrm{Id})^{-1}B$ is replaced by the advancement in time of (1) for a sufficiently long duration such that the time-harmonic solution is reached [6]. Care is taken so that the resulting eigenproblem remains Hermitian.

Figure $1(a)$ displays the optimal amplification as a function of the Strouhal number St for axisymmetric forcing in a case where the response is measured using the same norm as for the forcing. In figure $1(b, c)$, the corresponding optimal response is displayed.

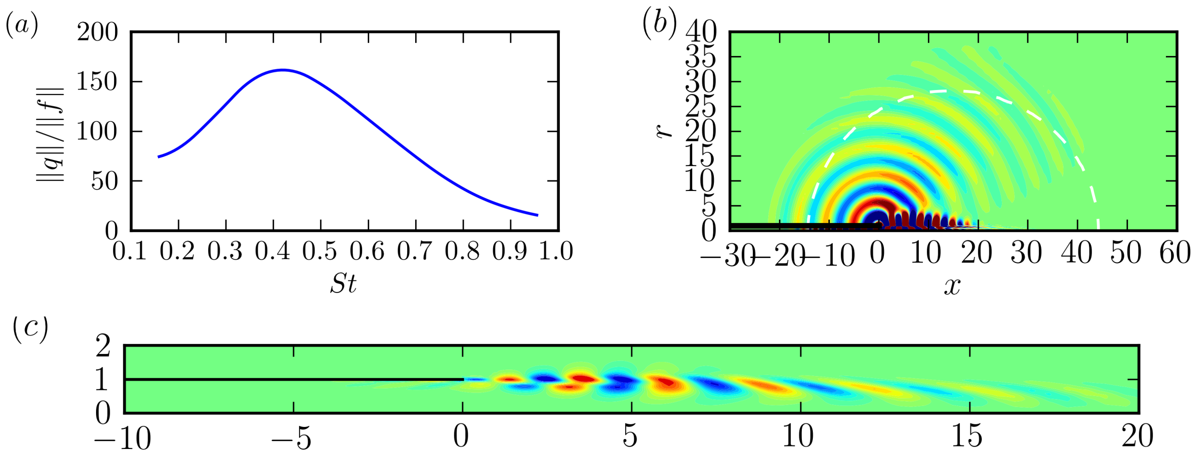

Figure 1. Results for axisymmetric forcing. (a): amplification factor as a function of the Strouhal number. (b, c): Dilatation and azimuthal vorticity field associated with the most efficient forcing.

CONCLUSIONS

An optimal forcing framework has been applied to the case of subsonic jets. This approach provides an efficient tool to describe convective instability mechanisms. The computed optimal Strouhal number for axisymmetric perturbations is in good agreement with experimental findings, and the corresponding response consists of vortices in the shear layer that reside within a few radii downstream of the potential core. Optimal excitation occurs when the forcing is located in the boundary layer close to the nozzle exit, with a shape suggestive of the Orr mechanism. The acoustic signature of these perturbations is characterized by preferred radiation at small angles with respect to the downstream jet axis.

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