DOUBLE DIFFUSIVITY ON MISCIBLE FLUID FLOW IN A CHANNEL

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Summary The pressure-driven miscible displacement of a less viscous fluid by a more viscous one in a horizontal channel is studied in which each fluid consists of a solution of two species diffusing at different rates and influencing the viscosity. The continuity and Navier-Stokes equations coupled to two convective-diffusion equations for the evolution of the concentration of both species are solved numerically. The result shows that double-diffusive (DD) effects can be destabilizing the classical miscible stable interface and demonstrate the development of various instability patterns. The intensity of the instability increases when increasing the diffusivity ratio between the faster-diffusing and the slower-diffusing species. This brings about fluid mixing and accelerates the displacement of the fluid originally filling the channel.

INTRODUCTION

Dynamics of interface patterns and mixing between two miscible fluids is an active research area that is of importance in many industrial processes, for example in enhanced oil recovery, fixed bed regeneration, hydrology and filtration. Displacement flows in porous media explain that if the displacing fluid is less viscous than the displaced one the interface separating them becomes unstable and fingering pattern develops at the interface. A review on such dynamics in porous media and Hele-Shaw cells is given in [1]. In a pipe flow, when a less viscous miscible fluid displaces a more viscous one, a two-layer/core-annular flow is obtained in most part of the channel/pipe as the elongated finger of the less viscous fluid penetrates into the bulk of the more viscous one. The interface between the two fluids becomes unstable forming Kelvin-Helmholtz (KH) instabilities and roll-up structures [2,3]. Recently Mishra et al [4] found that the classically stable situation of a more viscous fluid displacing a less viscous one in a porous media can be destabilized when two species diffusing at different rates and influencing the viscosity which affect the viscous fingering dynamics. Therefore, the main objective of this study is to understand whether such a concept of destabilization of an otherwise stable situation of two miscible fluids by differential diffusion effects can also occur in pressure-driven channel flows. Recently, an unstable double diffusive (DD) mode in a classically stable system of three-layer pressure-driven channel flow (with high and less viscous fluids occupying the core and near wall regions of the channel, respectively) has been found in [5] through a linear stability analysis.

MATHEMATICAL FORMULATION AND NUMERICAL SIMULATION

We consider a two-dimensional channel initially filled with a stationary, Newtonian incompressible fluid containing scalars S and F in quantity S_2 and F_2 and of viscosity μ_2. This solution is displaced by another miscible fluid where the scalars are present with a value S_1 and F_1 giving a viscosity μ_1 (see Fig. 1). The inlet fluid is injected with an average velocity, $V(\equiv Q/H)$, where Q and H denote the total flow rate and the height of the channel, respectively. We use a rectangular coordinate system (x, y) to model the flow dynamics, where x and y denote the horizontal and vertical coordinates, respectively. The channel inlet and outlet are located at $x = 0$ and L, respectively.

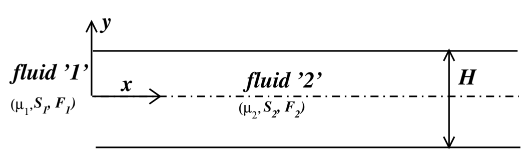

Figure 1. Schematic diagram the flow configuration: fluid '2' occupies the entire channel and is to be displaced by fluid '1'.

The flow dynamics are governed by the incompressible Navier-Stokes equations along with the convection-diffusion equations for the conservation of both species. They are non-dimensioqnalized with the time scale H^2/Q, length scale H and velocity with V and the dimensionless equations are

$$\nabla\cdot\mathbf{u}=0,\qquad \left[\frac{\partial\mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right]=-\nabla p+\frac{1}{\mathrm{Re}}\nabla\cdot\left[\mu(\nabla\mathbf{u}+\nabla\mathbf{u}^T)\right], \tag{1}$$

$$\frac{\partial s}{\partial t}+\mathbf{u}\cdot\nabla s=\frac{1}{\mathrm{ReSc}}\nabla^2 s,\qquad \frac{\partial f}{\partial t}+\mathbf{u}\cdot\nabla f=\frac{\delta}{\mathrm{ReSc}}\nabla^2 f, \tag{2}$$

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where $\mathbf{u} \equiv (u, v)$ is the velocity vector with components u and v in the x and y directions, respectively, p denotes pressure. s and f are two scalars, with f diffusing faster than s. Where $\mathrm{Re}(\equiv \rho Q/\mu_1)$ and $\mathrm{Sc}(\equiv \mu_1/\rho D_s)$ denote Reynolds and Schmidt numbers respectively. $\delta(\equiv D_f/D_s > 1)$ is the ratio of diffusion coefficients of the fast and the slow diffusing species. $\mathcal{D}_s$ and $\mathcal{D}_f$ are the diffusion coefficients of the S and F respectively. The dimensionless viscosity has the form $\mu = \exp{(R_s s + R_f f)}$, [4] where R_s $(\equiv (S_2 - S_1)d(\ln\mu)/dS)$ and R_f $(\equiv (F_2 - F_1)d(\ln\mu)/dF)$ are the log-mobility ratios of the species s and f, respectively.

These equations are solve using a finite volume method and discretized using a staggered grid. The scalar variables (the pressure and concentrations of the species) are defined at the center of each cell and the velocity components are defined at the cell faces. A weighted essentially non-oscillatory (WENO) scheme is used for the advective term and a central difference scheme is used to discretize the diffusive term similar to the method in [3].

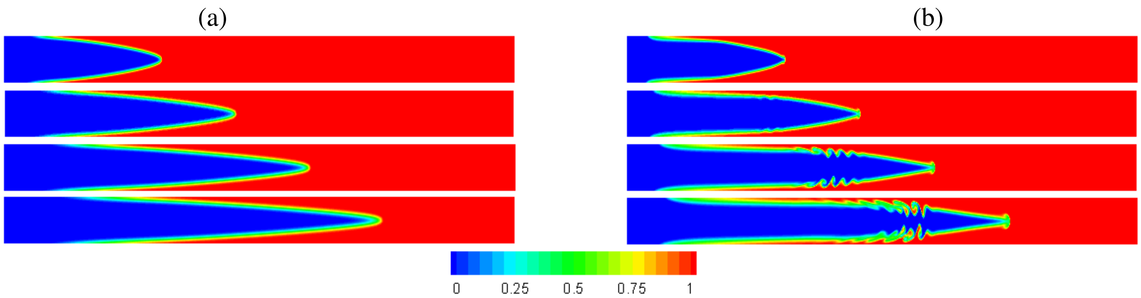

Figure 2. Spatio-temporal evolution of the concentration field of the species s at successive times (from top to bottom: $t = 20$, 30, 40 and 50) for (a) $\delta = 1$, and (b) $\delta = 10$. The rest of the parameter values are $Re = 100$, $Sc = 100$, $R_s = 3$ and $R_f = -3.6$.

Results and discussion

The spatio-temporal evolution of the concentration field of the species s for the parameter values $\mathrm{Re} = 100$, $\mathrm{Sc} = 100$, $R_s = 3$, $R_f = -3.6$ are plotted in Fig.2 for different values of δ. In the starting configuration at $t = 0$, the chosen log-mobility ratios correspond to a monotonically decreasing viscosity profile at the miscible interface of both fluids, which represents a classically stable interface. For $\delta = 1$, it can be seen that the high viscous fluid initially displaces the less viscous one like a miscible plug flow and then a fully-developed Poiseuille flow regime arises with a pure diffusive miscible interface. For $\delta > 1$ the miscible interface becomes unstable due to the double-diffusive (DD) mechanism. For a large $\delta(= 10)$ (see Fig. 2(b)) the flow becomes unstable forming symmetrical wavy interface which in turn produces the Kelvin-Helmholtz type instabilities. These instabilities occur at the interface primarily because a viscosity contrast arises because of the DD effects. At the tip of the single finger a "cap-type" instability is observed for the large value of δ $(= 10)$ (see Fig. 2(b)) unlike the spike-like instability for moderate value of $\delta = 5$ not shown here. This "cap-type" instability leads to a mushroom structure at vary late times. While looking the viscosity profile, it is observed that a high viscous stenosis region is formed due to the DD effects when $\delta > 1$ and such a region does not occur in the case $\delta = 1$. The displacement rate increases with increasing δ values.

CONCLUSIONS

We showed that due to the presence of DD effects a classically stable configuration of pressure-driven displacement flow of a less viscous fluid by a high viscous one becomes unstable. This brings about fluid mixing and accelerates the displacement of the fluid originally occupying the channel. Practically these types of instabilities can be obtained by considering the mass and heat as the slow and fast diffusing species, respectively. These results can be useful to find the non-reacting chemical species both influencing the viscosity of the solution and having different diffusion coefficients to predict such instability experimentally.

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