ANGULAR MOMENTUM TRANSPORT AND TURBULENCE IN LABORATORY MODELS OF KEPLERIAN FLOWS

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Summary Angular momentum transport in astrophysical flows is crucial for fundamental processes such as planetary formation and black hole accretion, although the mechanism is still not understood. We present angular momentum transport (torque) measurements for the turbulent flow of water between independently rotating cylinders. In addition to these recent studies, we reanalyze prior torque measurements to expand the range of control parameters for the experimental Taylor-Couette flows. We find that the torque may be described as a product of functions that depend only on the Reynolds number, which describes the turbulent driving intensity, and the rotation number, which characterizes the effects of global rotation. For a given Reynolds number, the global angular momentum transport for Keplerian-like flow profiles is approximately 14% of the maximum achievable transport rate. We argue that this level of transport from hydrodynamics alone could be significant in astrophysical settings.

INTRODUCTION

Rapidly-rotating shear flows are ubiquitous in geophysical and astrophysical settings such as planetary atmospheres, stellar interiors and accretion disks. In order for essential fundamental processes to occur, like matter inflow towards compact objects, there must be an exchange of angular momentum in such systems. Determining the flux of angular momentum in rotating shear flows is difficult and has been actively studied [1, 2] since the initial measurements of Wendt [3] and Taylor [4]. While shear tends to destabilize fluid flows, rotation provides linear stability whenever angular momentum increases radially outward ($dL/dr > 0$), which is known as the Rayleigh criterion; although, shear turbulence has been shown to occur even in Rayleigh-stable systems. Most astrophysical flows are Rayleigh-stable, which in the absence of other instabilities cannot transport angular momentum efficiently enough by viscous means alone to allow for the breadth of observed phenomena.

The question remains as to how astrophysical objects transport angular momentum at the observed rates. It is widely accepted that turbulence is mediated by an instability, which enhances angular momentum transport; although, the particular driving mechanism remains controversial. Here, we characterize the flux of angular momentum (torque) in Taylor-Couette flow between independently rotating cylinders as a function of the Reynolds number $Re = \frac{2}{1+\eta}|\eta Re_2 - Re_1|$ and the rotation number $R_\Omega = 1 - \eta(Re_1 + Re_2)/(\eta Re_2 - Re_1)$ [1], where ν is the kinematic viscosity, a (b) is the inner (outer) radius, $\eta = a/b$ is the radius ratio, and Re_1 (Re_2) is the inner (outer) cylinder Reynolds number. By achieving $Re > 10^6$ we observe that even Rayleigh-stable flow states that are similar to Keplerian flow profiles (so called quasi-Keplerian flows) become turbulent and transport angular momentum at astrophysically-relevant rates, which is in contrast to prior measurements [2]

RESULTS

We study the scaling of the dimensionless torque G over a range of Reynolds numbers for various constant rotation numbers (see Fig. 1), spanning a wide variety of flow states, including linearly stable cyclonic and anticyclonic flows as well as linearly unstable states. To expand the range of control parameters we also reanalyze prior torque measurements [3, 4, 1]. We find that the dimensionless torque G may be factorized as $G = f(R_\Omega)g(Re)$. To determine the effects of global rotation on the torque scaling, given by $f(R_\Omega)$, we normalize G for each Re by $G(Re, R_\Omega = 0)$ [1], which we denote as G_0. Our measurements of the normalized torque G/G_0 are shown along with prior measurements [3, 4, 1] in Fig. 1(a). The symbols correspond to measurements from distinct experiments, as described in the figure caption, while the solid line is the best fit for $f(R_\Omega)$. The function $g(Re)$ may then be determined by measuring the compensated torque $G/f(R_\Omega)$, which is shown in Fig. 1(b). In this case, the data from Maryland (black circles) and Twente (red squares), which each span a range of rotation numbers, are compared to prior measurements by Lewis and Swinney [5] (blue line) for a single rotation number corresponding to $\Omega_2 = 0$.

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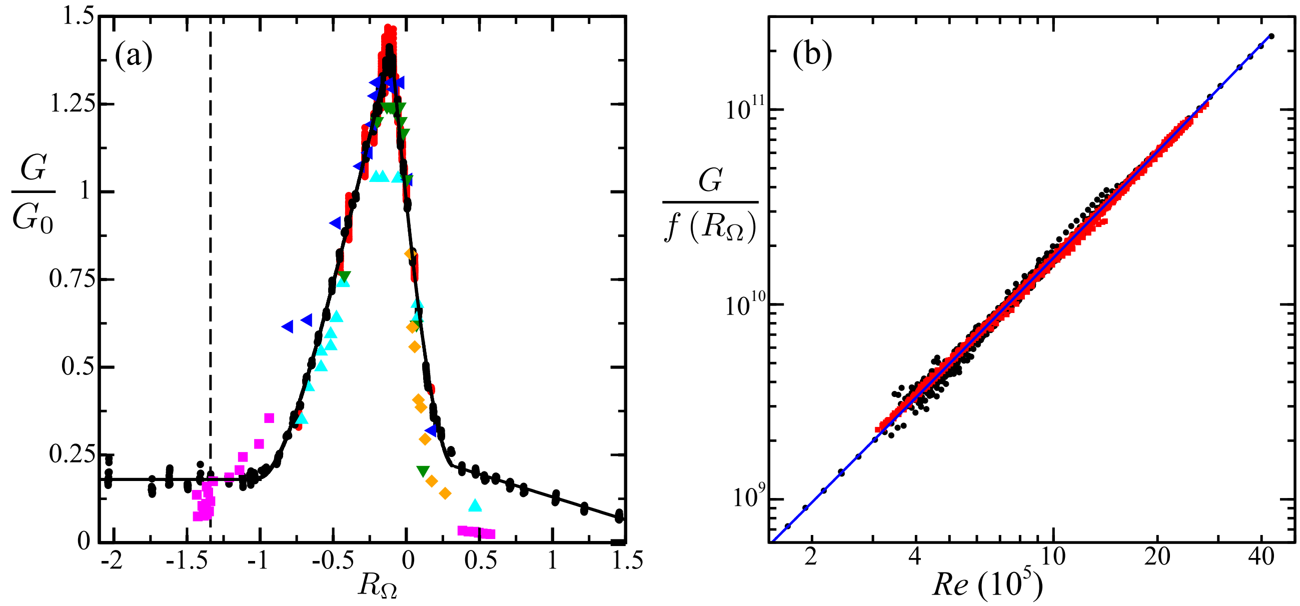

Figure 1. The effects of global rotation and turbulence intensity on the torque scaling in rotating shear flows may be determined by measuring (a) the normalized torque G/G_0 and (b) the compensated torque $G/f(R_\Omega)$. In (a) the data from Wendt [3] are shown as cyan ($\eta = 0.68$), blue ($\eta = 0.85$) and green ($\eta = 0.935$) triangles. Additional radius ratios are examined using the data from Taylor [4], shown as orange diamonds, Richard [1], given by magenta squares, Maryland [6] indicated by black circles and Twente [7] by red circles. The solid line is the best fit for $f(R_\Omega)$. Keplerian flow states are indicated by the dashed line at $R_\Omega = -4/3$. In (b) we compare measurements of the compensated torque $G/f(R_\Omega)$ for the entire range of R_Ω explored by Maryland (black circles) and Twente (red squares) to the best fit from Lewis and Swinney [5] for the case $\Omega_2 = 0$.

CONCLUSIONS

The measurements that we present here indicate that the dimensionless torque G may be described as a product of functions that separately depend upon the Reynolds number Re and the rotation number R_Ω. The rotation number R_Ω introduced by Dubrulle *et al.* [1] collapses all of the experimental data presented here for various radius ratios, aspect ratios and Reynolds numbers. The effects of global rotation, measured by G/G_0, are shown to be well-described by the function $f(R_\Omega)$ for the entire range of R_Ω explored in the present studies. Furthermore, we observe that the torque for quasi-Keplerian flows is 14% of the maximum achievable value for a given Reynolds number. We argue that this level of angular momentum transport may be significant in astrophysical flows, in contrast to prior measurements and conclusions.

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