HYDRODYNAMIC INSTABILITY OF THE SPIRAL FLOW

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Summary The instability of spiral flow is investigated numerically. Systematically studied the effect of the inner cylinder rotation and ratio of the inner and outer cylinders diameter. The neutral stability curves and the increment of the disturbances growth are presented. The obtained numerical results are compared with known experimental and theoretical data.

In different technological problems, the flow between two cylinders is to be created. Such a problem arises while driling of boreholes, in conveyer screw problems, in heat exchange devices, in lubrication systems. Conditions of the laminar-turbulent transition in the flow may have a significant impact on efficiency of a technological solution. The spiral flow in cylindrical annulus is the superposition of two model flows: cylindrical Poiseuille and Couette flows. The laminar-turbulent transition in overwhelming majority model flows (boundary layer on the plate, plane Poiseuille flow, cylindrical Couette flow, etc.) has been studied well. However up to now we have very small information about the hydrodynamical instability of forced cylindrical flow and spiral flow. There are three experimental works [1 - 3] and some attempts of the numerical investigations (see papers [4–7]). Thus even linear stage of the instability development in spiral flows has not been investigated. The study of the linear stage of the laminar-turbulent transition in a cylindrical annulus with the rotation of the inner cylinder is the purpose of this article.
It was considered the flow between two coaxial cylinders of radii R_i for inner and R_o for outer cylinder. Rotation rate of inner cylinder is Ω . Studied the effect on the stability characteristics radii ratio $\theta = R_i/R_o$ and the axial $\mathrm{Re}_z = W_{max}(R_o - R_i)/2\nu$ and azimuthal $\mathrm{Re}_\varphi = \Omega R_i(R_o - R_i)/2\nu$ Reynolds numbers. Here W_{max} is the scale of axial velocity, ν is kinematic viscosity. The disturbances decomposed on normal modes $exp(i\alpha z + in\varphi - i\omega t)$ with axial wave number α, azimuthal wave number n and complex frequency ω. Eigenvalue problem for linearized Navier – Stokes equations solved numerically by Galerkin method [8, 9].
Neutral stability curves constructed for flow parameters used in experiments [2] are presented on figure 1. Experimental points shown on curves as diamonds. Behaviour of the neutral curves demonstrate coexistence of the two different mechanisms of instability in this diapason of Re_φ (see table 1). The first one is the instability of the axial Poiseuille flow arising for sufficiently large axial Reynolds number (curve 1). Curves 2 - 6 on figure 1 correspond to instability excited by inner cylinder rotation. In this case the form of neutral curves, azimuthal wave numbers changes rapidly with variation of Re_φ.

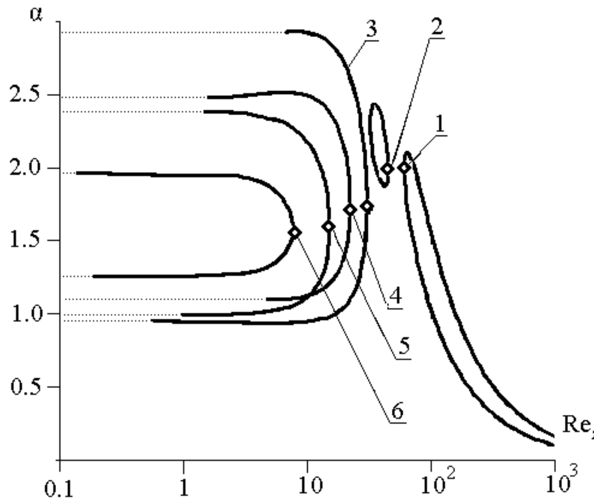

Fig. 1. Neutral stability curves for different azimuthal modes in gap $\theta = 0.5$

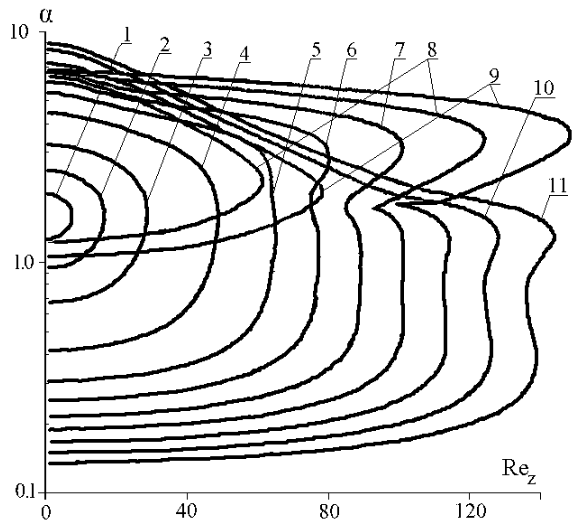

Fig. 2. Neutral stability curves for different azimuthal Reynolds numbers in gap $\theta = 0.5$

On figure 2 are presented neutral stability curves for the same flow, but in more wide diapason of variation of Re_φ. Disturbances of the azimuthal mode $n = 0$ is considered, similar to curves 5, 6 on figure 1. Easy to see that this kind of instability is typical for the flow and the area of flow instability bounded by neutral curves growth with increasing of Re_φ. For sufficiently large values of $\mathrm{Re}_\varphi > 100$ obtained nonmonotonic branches on the neutral curves as on curves 6 – 11. Neutral curves 8, 9 demonstrate the existence of special diapason of the Re_φ values corresponding to branching of neutral curves which disappear with growth of Re_φ.

Neutral curves on figures 3, 4 demonstrate instabilities of the flows in wide $(\theta = 0.2)$ and narrow $(\theta = 0.8)$ gaps. It is qualitative similarity between this curves and neutral curves for the flow with $\theta = 0.5$. The areas of instability expand with increasing of Re_φ. Nonmonotonic behavior in wide gap (curves 6, 7 on fig. 3) occurs for the same diapason of values of $\mathrm{Re}_\varphi \approx 150 \div 180$ as for middle gap, but the branching of neutral curves not occur in this case. For the flows in narrow gap the diapason of azimuthal Reynolds values leading to nonmonotonic and branching expands to at least $\mathrm{Re}_\varphi \approx 150 \div 300$.
This results can conclude that known experimental data [2] is the demonstration by some special instability effects, which have no theoretical explanation for a while. First possible assumption is the influence of spatial curvature of the velocity field combined from Poiseuille and Couette flows on stability. Quantities of axial Reynolds numbers and axial wave numbers point at essential contribution of the Couette flow components in this effects, but detail analysis of the problem demands additional efforts. Results of this work can serve as a challenge for choosing a regular path on the modelling of transition in spiral flows.

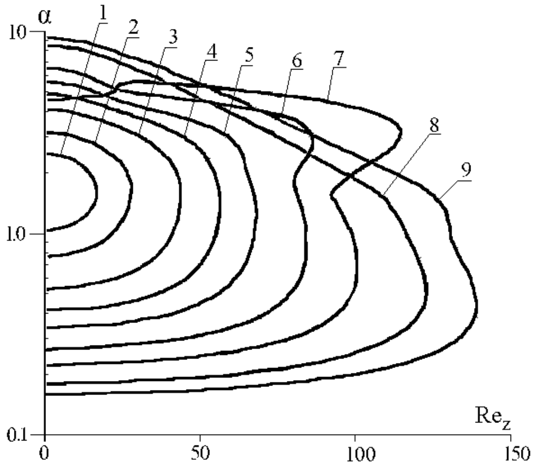

Fig. 3. Neutral stability curves for the spiral flow in gap $\theta = 0.2$

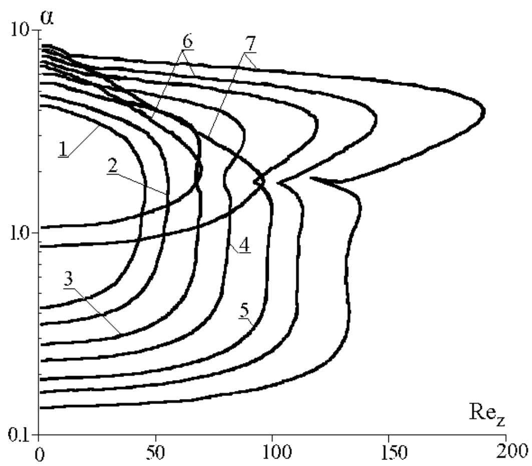

Fig. 4. Neutral stability curves for the spiral flow in gap $\theta = 0.8$

Table 1. Azimuthal Reynolds numbers for curves on figures 1 – 4.

Curve number	1	2	3	4	5	6	7	8	9	10	11
n (fig. 1)	5	3	1	1	0	0					
Re_φ (fig. 1)	51	52	48.5	43	38.92	35.33					
Re_φ (fig. 2)	35.33	40	50.425	75	100	120	140	160	180	200	220
Re_φ (fig. 3)	50	60	80	100	120	150	180	220	250		
Re_φ (fig. 4)	100	120	150	180	220	250	300				

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