ON THE DUAL ROLES OF VISCOSITY TO THE FLOW INSTABILITY

Hua-Shu Dou[*a)] , Boo Cheong Khoo[**]
[*]*Faculty of Mechanical Engineering and Automation, Zhejiang Sci-Tech University, Hangzhou, Zhejiang 310018, China*
[**]*Department of Mechanical Engineering, National University of Singapore, Singapore 119260*

Summary The energy gradient method is successfully used to analyze the role of viscosity to flow stability. It is found that under finite disturbance viscosity has only stabilizing role in parallel flows and circular flows, and viscosity has both stabilizing role and destabilizing role in boundary layer flow. The stable role is from the damping of the viscous friction to the disturbance. The unstable role is due to the diffusion of the transverse velocity. These results agree well with the experimental observations.

INTRODUCTION

In previous studies, it was found that viscosity may play dual role to flow stability in some flows [1]. However, this kind of dual role is not observed in pipe flow, plane Couette flow and Taylor-Couette flow [1]. This problem needs to be clarified in order to have a better understanding on the physics of turbulent transition and flow instabilities. Recently, Dou and co-authors [2-5] proposed an energy gradient method (EGM) with the aim of clarifying the mechanism of flow transition from laminar flow to turbulence. For plane Poiseuille flow and pipe Poiseuille flow (see Fig. 1), this method yields consistent results with experimental data. It also obtained agreement with experiments for primary instability in Taylor-Couette flows [4]. In this study, the dual roles of the viscosity to flow stability are analyzed with the EGM. The physics of how viscosity affects the flow stability under finite disturbance is demonstrated.

ENERGY GRADIENT METHOD

When a disturbed fluid particle moves around a streamline, this particle will gain energy from the base flow due to the non-uniformity of the flow, ΔE , and lose energy due to viscous friction, ΔH (Fig.2). The stability of a flow depends on the relative magnitude of ΔE and ΔH . For flow with a curved streamline, the relative magnitude of the energy gained from collision and the energy loss due to viscous friction determines the disturbance amplification or decay. Thus, for a given flow, a stability criterion with finite disturbance can be written as follow for a half-period [2-5],

$$F = \frac{\Delta E}{\Delta H} = \left(\frac{\partial E}{\partial n} \frac{2A}{\pi} \right) \Big/ \left(\frac{\partial H}{\partial s} \frac{\pi}{\omega} u \right) = \frac{2}{\pi^2} K \frac{A\omega}{u} = \frac{2}{\pi^2} K \frac{v'_m}{u} < Const \, , \tag{1}$$

and

$$K = \frac{\partial E / \partial n}{\partial H / \partial s} . \tag{2}$$

Here, F is a function of coordinates which expresses the ratio of the energy gained by the particle in a half-period and the energy loss due to viscosity in the half-period. K is a dimensionless field variable (function) and expresses the ratio of transversal energy gradient and the rate of the energy loss along the streamline, $E = p + (1/2)\rho V^2$ is the total mechanical energy, s is along the streamwise direction and n is along the transverse direction. It can be seen that the larger the value of K, the more unstable the flow, for same amplitude of finite disturbance.

ANALYSIS OF ROLE OF VISCOSITY IN VARIOUS FLOWS

The calculation of $\partial H / \partial s$ can be done using the principle of energy conservation. Generally, the energy loss of unit volumetric fluid along the streamwise direction equals to the derivatives of the total mechanical energy in the streamwise directions plus the work done to the fluid by the external [4],

$$\frac{\partial H}{\partial s} = -\frac{\partial E}{\partial s} + \frac{\partial W}{\partial s} , \tag{3}$$

where W is the work done to the unit volume fluid by the external. For pressure driven flows, $\mathrm{W} \equiv 0$.

[a)] Corresponding author. Email: huashudou@yahoo.com.

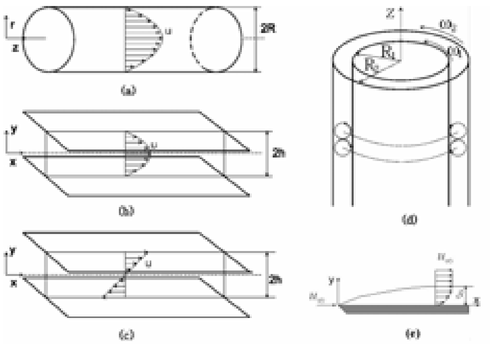

Fig.1 Schematic of wall bounded parallel flows. (a) Pipe Poiseuille flow; (b) Plane Poiseuille flow; (c) Plane Couette flow; (d) Taylor-Couette Flow; (e) Boundary layer flow.

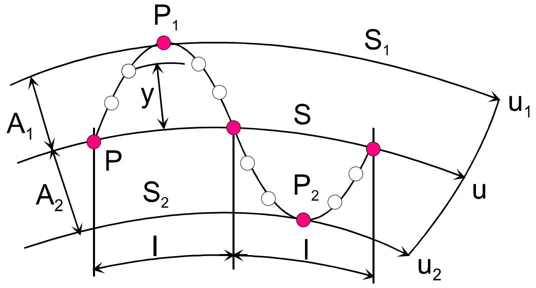

Fig.2 Movement of a particle around its original equilibrium position in a cycle of disturbance for curved flows.

The derivative of the total mechanical energy in the transverse direction, $\partial E/\partial n$, in above equation can be calculated by the equation of incompressible Navier-Stokes equation, $\rho(\partial \mathrm{u}/\partial \mathrm{t}) + \nabla(p + 0.5\rho u^2) = \mu\nabla^2\mathrm{u} + \rho(\mathrm{u}\times\nabla\times\mathrm{u})$. Thus,

$$\frac{\partial E}{\partial n} = \frac{\partial(p + (1/2)\rho u^2)}{\partial n} = \rho(\mathrm{u}\times\omega)\cdot\frac{\mathrm{dn}}{|\mathrm{dn}|} + (\mu\nabla^2\mathrm{u})\cdot\frac{\mathrm{dn}}{|\mathrm{dn}|} = \rho u\omega + (\mu\nabla^2\mathrm{u})_n, \qquad (4)$$

where $\omega = \nabla\times\mathrm{u}$ is the vorticity. The magnitude of the K function can be obtained as,

$$K = \frac{\partial E/\partial n}{\partial H/\partial s} = \frac{\rho u\omega + (\mu\nabla^2\mathrm{u})_n}{\partial H/\partial s} = \frac{\rho u\omega}{\partial H/\partial s} + \frac{(\mu\nabla^2\mathrm{u})_n}{\partial H/\partial s}. \qquad (5)$$

For pressure driven flows, $\partial H/\partial s = (\mu\nabla^2\mathrm{u})_s$, we have $K = \frac{\rho u\omega + (\mu\nabla^2\mathrm{u})_n}{(\mu\nabla^2\mathrm{u})_s} = \frac{\rho u\omega}{(\mu\nabla^2\mathrm{u})_s} + \frac{(\mu\nabla^2\mathrm{u})_n}{(\mu\nabla^2\mathrm{u})_s}$. Further, for pressure driven parallel flows (pipe and plane Poiseuille flows), $(\mu\nabla^2\mathrm{u})_n = 0$, $K = \rho u\omega/(\mu\nabla^2\mathrm{u})_s$. The value of K is reduced with increasing viscosity. Thus, viscosity has only stable role to the pressure driven parallel flow. For shear driven parallel flows (plane Couette flow), $(\mu\nabla^2\mathrm{u})_n = 0$, and $\partial H/\partial s = \frac{\tau}{u}\frac{du}{dy} = \frac{1}{u}\mu(\frac{du}{dy})^2$. Equation (5) becomes, $K = \frac{\rho u\omega}{\partial H/\partial s} = \rho u\omega/\left[\frac{1}{u}\mu(\frac{du}{dy})^2\right]$, which shows that the value of K decreases with increasing viscosity. Thus, viscosity has only stable role to the flow. For Taylor-Couette flow, $(\mu\nabla^2\mathrm{u})_n = 0$, we have $K = \frac{\rho u\omega}{\partial H/\partial s} = \rho u\omega/\left[\frac{\tau}{u}\frac{du}{dr} + \frac{\tau}{r}\right]$. Since the shear stress τ is proportional to the viscosity μ, the value of K is reduced with increasing viscosity. Thus, viscosity has only stable role to the flow. For boundary layer flow, $K = \frac{\rho u\omega}{(\mu\nabla^2\mathrm{u})_s} + \frac{(\mu\nabla^2\mathrm{u})_n}{(\mu\nabla^2\mathrm{u})_s}$. In this equation, $(\mu\nabla^2\mathrm{u})_n \neq 0$, which causes viscous diffusion while $(\mu\nabla^2\mathrm{u})_s$ leads to energy dissipation. The former has unstable role and the latter has stable role. Therefore, in boundary layer flow, viscosity has dual roles to the flow stability.

CONCLUSIONS

The energy gradient method is successfully used to analyze the roles of viscosity to flow stability under finite disturbance. It is found that viscosity has only stabilizing role in parallel flows and circular flows, and viscosity has both stabilizing role and destabilizing role in boundary layer flow. The stable role is from the damping of the viscous friction to the disturbance. The unstable role is due to the viscous diffusion of the transverse velocity. These results agree well with the experimental observations.

References

[1] Drazin P.G., Reid W.H.: *Hydrodynamic Stability*, Cambridge University Press, Cambridge, England, 1981.
[2] Dou H.-S.: Mechanism of flow instability and transition to turbulence, *Inter. J. Non-Linear Mech.*, **41:** 512-517, 2006.
[3] Dou H.-S.: Physics of flow instability and turbulent transition in shear flows, *Inter. J. Phys. Sci.*, **6**(6): 1411-1425, 2011.
[4] Dou H.-S., Khoo B.C., Yeo K.S.: Instability of Taylor-Couette flow between concentric rotating cylinders, *Inter. J. of Therm. Sci.*, **47:** 1422-1435, 2008.
[5] Dou H.-S., Khoo B.C.: Mechanism of wall turbulence in boundary layer flows, *Modern Physics Letters B*, **23:** 457-460, 2009.