GENERAL MECHANISM FOR THE MEANDERING INSTABILITY OF RIVULETS OF NEWTONIAN FLUIDS

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Summary Through controlled excitation, we show that the meandering instability of a rivulet is linear and convective near the threshold, and that it results from two key ingredients: fluid inertia and anisotropy of the friction between rivulet and substrate. Meandering only occurs if the motion normal to the instantaneous flow path is more difficult than parallel to it. A criterion for the onset of meandering derived from these ingredients agrees quantitatively with the experiment. Other instabilities of thin liquid filaments and sheets (buckling of impacting liquids, oscillation of high-speed ejected sheets) can also be reviewed in the light of this generic instability mechanism.

INTRODUCTION

Meandering of rivulets down a vertical or inclined surface is a well known phenomenon[1], which is a fundamental ingredient to our understanding of flow over solid surfaces. For example, the meandering pattern determines the degree to which a solid surface is covered by a liquid film: the width of a rivulet may be 1 mm, while the transversal excursions of a meander may be several cm. The small-scale meanders discussed in this paper also fall into a broader class of instabilities which include river meandering [2] and the coiling of fluid jets [3]. Most explanations of meandering are based on the idea that centrifugal forces amplify the curved parts of the stream, while others emphasize the role of noise and surface disorder. However, as yet there exists no quantitative theory of meandering; indeed, it is unclear how centrifugal forces can generate instability. A straight rivulet appears to be more favorable than a curved one, both from a point of view of an inertial mass, as well as from considerations of surface energy.

EXPERIMENT

PDMS oil is injected between two vertical glass panes forming a Hele-Shaw cell with a spacing b (between 0.5 and 2 mm). The liquid wets the glass, and is injected in such a way as to form a ridge linking both glass surfaces.
At low flow rate the rivulet is perfectly straight and has a constant width. Above a critical flow rate, meandering patterns begin to develop, which are irregular in general, and which are convected downstream, as shown in the figures below.

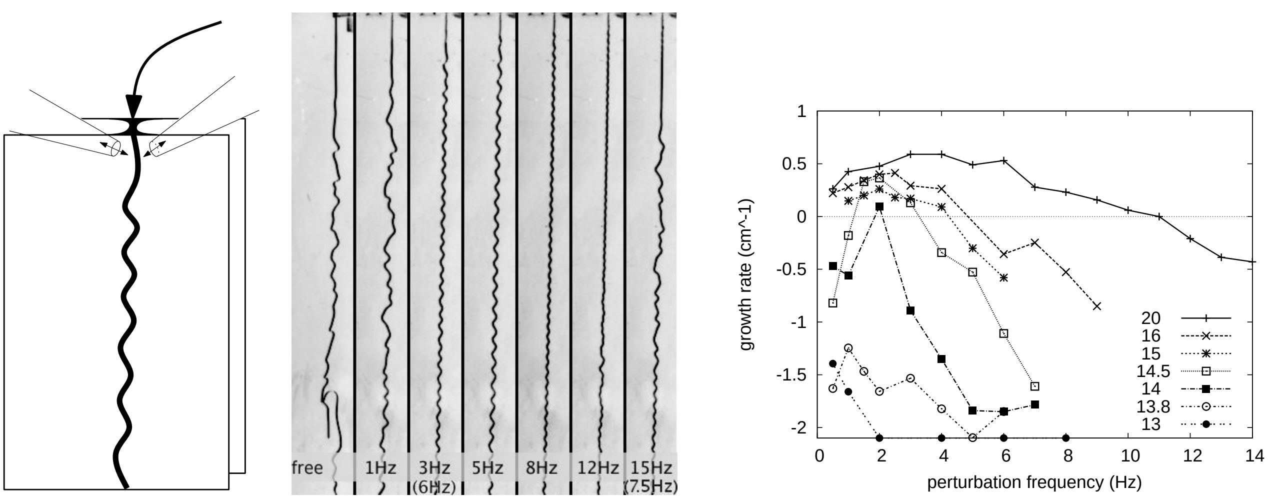

FIG 1. (left) A rivulet injected between two glass plates can be perturbed laterally by nozzles blowing and sucking air out of phase. **(middle)** Without perturbation, the rivulet exhibits a meandering instability above a critical flow rate (free regime); a perturbation at a fixed frequency selects certain modes of instability, sometimes through period- or frequency-doubling. **(right)** From the initial growth rate of controlled perturbations we obtain the dispersion relation of the system at different flow rates (given in cm^3/min), which is reminiscent of a linear instability. The most unstable frequency at threshold is around 2 Hz.

From the initial growth rate of excitations at different frequencies (Fig. 1), we can construct the dispersion relation of the rivulet. It shows that growth rates become positive for the first time at the critical flow rate (13.8 cm^3/min) and for a 2 Hz perturbation. At higher flow rates both the growth rate and the range of unstable frequencies increase.

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We can also measure the phase velocity from these visualisations, and the fluid velocity from close-up measurements (Fig. 2). The fluid velocity is always above the phase velocity, so that the fluid is moving along a curved, constrained path.

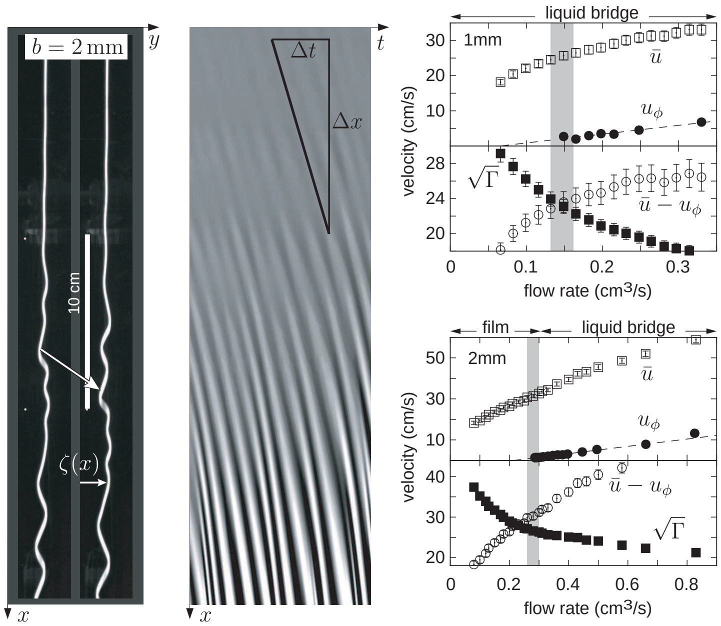

FIG. 2 (left) Images of a meander showing downstream convection. **(middle)** Spatio-temporal representation of the meander on the left, where the gray level is proportional to th local deviation from a straight path. **(right)** Comparison of the velocity and capillary terms appearing in equ. (2) from experimental data. The grey region show the observed meandering threshold, where the terms $\bar{u} - u_\phi$ and $\sqrt{\Gamma}$ should be equal according to our instability criterion (2).

MODEL

The depth-averaged Navier-Stokes equations for the rivulet lead to the following equation for the velocity field $\mathbf{u} = (u, v)$ in the (x, y) plane of the cell:

$$\partial_t \mathbf{u} + \beta(\mathbf{u} \cdot \nabla)\mathbf{u} = -k\mathbf{u} + (\Gamma\kappa - k_{cl} u_n)\mathbf{n} + g\mathbf{e}_x. \tag{1}$$

Here Γ is the effective line tension of the rivulet, and $k = 12\nu/b^2$ a Poiseuille flow friction factor (ν the kinematic viscosity), g the gravitational acceleration and $\beta \approx 1$ a constant. The factor $k_{cl} \propto k\sqrt{b/h}$ describes the additional "contact line" friction which arises when the rivulet possesses a nonzero velocity u_n normal to itself (h is the surrounding film thickess). This term proves crucial to the instability, which a linear perturbation analysis shows to occur when

$$(\bar{u} - u_\phi)^2 > \Gamma, \tag{2}$$

where $\bar{u}$ is the mean velocity and $u_\phi = \frac{k}{k+k_{cl}}\bar{u}$ the phase velocity. All terms of the criterion (2) can be measured experimentally, and we find a good quantitative agreement, without adjustable parameters (Fig. 2(right), and ref [4]).

DISCUSSION

The instability criterion (2) shows that meandering in our set-up occurs when inertial forces (left side) exceed the stabilising line tension Γ. The relevant velocity turns out to be the difference between material velocity and phase velocity. This can be understood if one realises that the rivulet path acts as a constraint on the motion. This is the case because shifting the rivulet path is hindered by an additional friction, namely the "contact line" friction term containing k_{cl} in equ. (1), that slows its motion with respect to the liquid. For a constrained motion along a curved path inertial forces act in a centrifugal, amplifing sense. The path provides the relevant reference frame to evaluate the centrifugal force, meaning the phase velocity has to be subtracted from the liquid velocity. This reasoning can be applied to other instabilities[4, 3].

References

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