DIRECT NUMERICAL SIMULATION OF RIGID PARTICLE-FREE SURFACE INTERACTION

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Summary An arbitrary-Lagrangian-Eulerian (ALE) approach is developed for the direct numerical simulation of the rigid particle-free surface interaction problem. The rigid particle is accelerated by an external force toward the free surface to achieve certain impact velocities. We find that the minimum film thickness between the rigid particle and free surface occurs after the particle reaches its highest position. At high impact velocities, the film thickness eventually levels off due to film drainage and the thickness is not sensitive to impact velocities.

INTRODUCTION

The interactions between particles and bubbles are important in many applications, such as mining[1] and energy industries. This process is very complicated and has three stages[2, 3]. In the fist stage, the distance between the particle surface and the free surface is large. The particle motion is determined by hydrodynamic forces only. When the distance gets small, a film forms and the film drainage occurs, which is known as the second stage. As the distance further decreases, the film becomes unstable and eventually ruptures, which is the third stage.

Different models have been proposed to predict the bubble-particle collision rate and film rupture during these three stages [4]. In comparison, the direct numerical simulation, which can provide a better understanding of the problem, is rarely explored due to its difficulties. Firstly, the motion of the rigid particle and that of the fluid should be fully coupled. Secondly, the deformation of free surface should be considered. Thirdly, the thin film should be resolved by the computational mesh in order to predict the film drainage reliably. Fixed mesh methods, eg., Finally, different factors such as van der Waals force and surface roughness should be considered to trigger the film rupture. For now, we only want to study the first two stages of the particle-free surface interaction, therefore we can leave the film rupture problem for future work. The ALE method based on a mesh that moves with particles is well suited for particle flows[5, 6]. Fixed-mesh based methods, such as volume-of-fluid[7], level-set[8], front-tracking[9], and phase-field[10], can be used to track the moving interface. However, in order to resolve the think film at the same time, the ALE method is still the first choice[11]. In this work, we develop an ALE method to track the motions of particles and free surfaces and to capture the film drainage as well.

NUMERICAL METHODS

We consider the interaction between an upward moving particle and a horizontal free surface, which can be treated as axisymmetric. For the fluid, the governing equations include the continuity equation and the momentum equation

$$\nabla \cdot \boldsymbol{u} = 0, \quad \rho(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}) = \rho \boldsymbol{f} + \nabla \cdot \sigma, \tag{1}$$

where ρ, $\boldsymbol{u}$, t, σ and $\boldsymbol{f}$ are fluid density, velocity, time, stress tensor and body force, respectively. Due to axisymmetry, the particle has no agular motion and the translational motion is governed by the Newton's second law of motion,

$$m_p \frac{d\boldsymbol{U_p}}{dt} = \boldsymbol{G_p} + \boldsymbol{F_p} = \boldsymbol{G_p} - \int_{\partial\Omega_p(t)} \sigma \cdot \boldsymbol{n}\, dS, \tag{2}$$

where $\partial\Omega_p(t)$ denotes the boundary of the domain occupied by the rigid particle at time t, m_p is the particle mass and $\boldsymbol{U_p}$ is the particle velocity, $\boldsymbol{G_p}$ is the body force, $\boldsymbol{F_p}$ is the hydrodynamic force, and $\boldsymbol{n}$ is the normal to boundary that points out of the fluid. On the free surface, a stress condition based on the Young-Laplace equation is imposed. The no-slip boundary condition is imposed on the particle surface.

The governing equations 1-2 are solved in a finite element framework on a triangular moving mesh. The second-order implicit schemes are used for temporal discretization. Mesh is updated according to the velocities of particle and free surface. Within each time step, iterations between fluid flow and mesh motion are performed to enhance stability.

RESULTS AND DISCUSSIONS

We consider the axisymmetric problem with y axis as the symmetry axis, as shown in Fig. 1(a). The rigid particle is initially at rest and then accelerated by an external force until it reaches the position 5 particle radii below the undisturbed free surface. We define the particle velocity at this position as the "impact velocity". All parameters are non-dimensionalized by particle radius, fluid density and viscosity.

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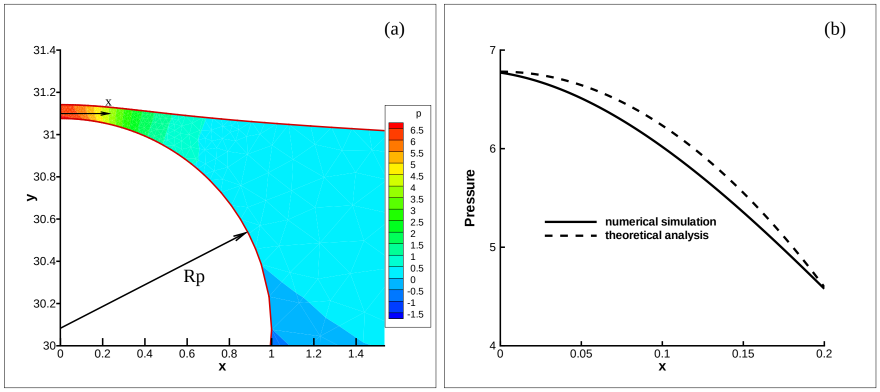

Figure 1. (a) Pressure contours and (b) Comparison with the lubrication theory.

When the rigid particle approaches a free surface, a film forms between the particle and free surface. Figure 1(a) shows a typical pressure distribution when the particle almost reaches its highest position. At this time, the film thickness is thin enough to apply the lubrication theory, which is confirmed by the comparison of pressure inside film, as shown in Fig. 1(b).

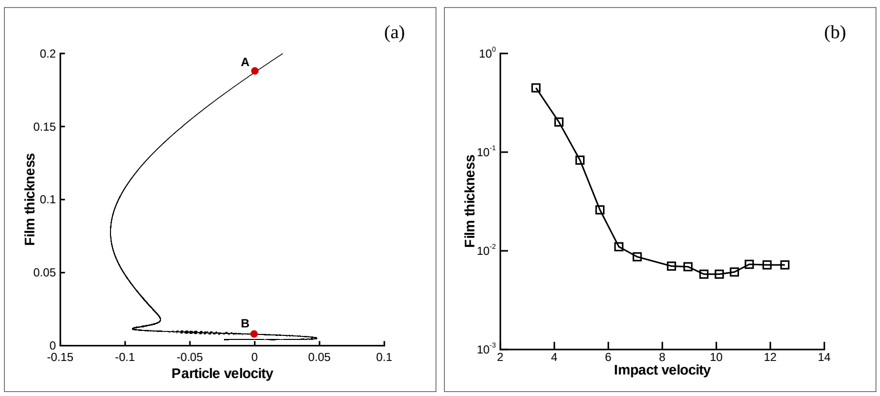

Figure 2. (a)Film thickness vs particle velocity and (b) Minimum film thickness as a function of impact velocity.

Figure .2(a) shows the film thickness vs particle velocity for the case of impact velocity 7.7. At point A, the particle velocity decreases to zero and the particle gets to its highest position. However, the film thickness is still decreasing as the particle has a negative acceleration until point B, the film thickness almost reaches the minimum value. We plot the minimum film thickness as a function of impact velocity in Fig. 2(b). It can be found that the film thicknesses decreases and approaches a constant value with the increasing of impact velocity.

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