# RHEOLOGICAL PROPERTIES OF BUBBLE SUSPENSIONS IN YIELD STRESS FLUIDS

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*Summary* We study experimentally and theoretically the behavior of suspensions of bubbles dispersed in a yield stress fluid. We observed that the overall properties of bubble suspensions depend upon the deformability of the bubbles. For deformable bubbles, the yield stress of the suspension is the same as the yield stress of the suspending fluid and the overall elastic modulus is a decreasing function of the bubble volume fraction. For non-deformable bubbles, the yield stress and the elastic modulus of the suspension increase with the bubble volume fraction. Comparison of experimental data with theoretical modeling shows that the capillary number accounting for the deformability of the bubbles is not the same in the solid-elastic regime and in the liquid-plastic regime: deformability of the suspending fluid must be accounted for by the elastic shear modulus in the elastic regime whereas bubbles deformability should not be taken into account in the plastic regime.

## INTRODUCTION

Air bubbles are incorporated into yield stress fluids in many situations of practical interest (plaster board and cellular lightweight concrete manufacturing processes, lightweight thermal insulation mortars, food foams and mousses, ...). Even if primary goal for adding air to a material is to modify its properties in the hardened state, fluid material workability also depends upon gaseous phase properties as bubble size, air-paste surface tension, bubble volume fraction. Then, one needs to study the air effects on the fluid overall rheological properties in order to analyze and to improve handling and placing processes.

## MATERIALS AND METHODS

Well controlled suspensions of monodisperse bubbles in a yield stress fluid are prepared thanks to an original method that consists in mixing a foam and an oil-in-water emulsion. The overall elastic modulus of the suspension have been measured through oscillatory shear experiments in the linear regime A wide gap geometry is used to ensure that there are enough bubbles in between the inner and the outer tools so that we measured the properties of a continuum material (the bubble supension). We measured the yield stress $\tau_c$ through a single measurement at a small constant strain rate using the same vane in cup geometry. The yield stress is defined as the shear rate plateau in a shear stress vs shear rate plot.

Two different results are found depending on the size (thus the deformability) of the bubbles. For deformable bubbles (ie large bubbles), the yield stress of the suspension of bubbles is found to be the same as the yield stress of the suspending fluid, while the elastic modulus decreases. For non-deformable bubbles (ie small bubbles), both the yield stress and the elastic modulus of the suspension are found to increase with the bubble volume fraction.

## ELASTIC MODULUS

Experimental data for the elastic modulus of a monodisperse suspension of bubbles with radius 150 $\mu$m are depicted in the left diagram in Figure 1. We observed that the shear modulus $G'(\varphi)$ is a decreasing function of the bubble volume fraction $\varphi$. To account for the relative stiffness of the bubbles compared to that of the ambient fluid, it is convenient to introduce the capillary number $C_a = (G'(0) \times R)/\sigma_t$ with $G'(0)$ the matrix elastic modulus, $R$ the bubble radius and $\sigma_t$ the surface tension in the air-matrix inferface. Experimental elastic modulus data depicted in Figure 1 correspond to a suspension with $C_a = 3$. When the capillary number is naught, the bubbles remain spherical while they do not resist to shear straining when the capillary number tends toward infinity. Lower and upper bounds for the overall elastic modulus of a dilute suspensions of bubbles are plotted in the left diagram in Figure 1. The lower bound (blue straight line) corresponds to an infinite capillary number while the upper bound (red straight line) corresponds to a naught capillary number. Satisfactorily, our experimental data lie in between the two bounds.

Palierne equation allows to connect the overall elasticity modulus of a dilute emulsion with the elastic modulus of the two components (the ambient fluid and the droplet fluid), the size of the droplets and the air-paste surface tension. When the elastic modulus of the droplets is naught, the Palierne equation reads

$$G'(\varphi) = G'(0) \times \left(1 + \varphi \frac{1 - 2C_a}{1 + 5/6 C_a}\right) \qquad (1)$$

The black straight line plotted in the left diagram in Figure 1 is the Palierne estimate with $C_a = 3$. The Palierne estimate was obtained assuming that the bubble volume fraction $\varphi$ is small (dilute suspension). Surprisingly equation (1) accurately fits the experimental data for values of the bubble volume fraction much larger than expected. For a given value of the bubble volume fraction, the Palierne equation (1) predicts that the dimensionless elastic modulus is a decreasing function of the bubbles radius. This theoretical prediction is consistent with our experimental data.

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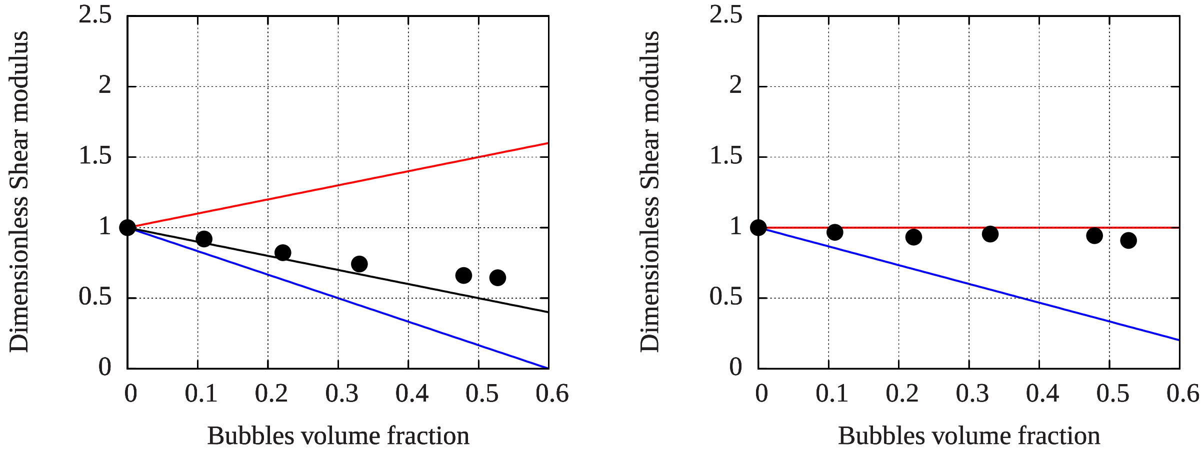


**Figure 1.** The dimensionless shear modulus (left diagram) and the dimensionless yield stress (right diagram) of a suspension of monodisperse 150 $\mu$m bubbles dispersed in a yield stress fluid. The red straight line in the left diagram (respectively in the right diagram) is the Palierne estimate (1) (respectively the overall yield stress estimates) with $C_a = 0$. The black straight lines and the red straight lines are the same estimates with $C_a = 3$ and $C_a \to \infty$.

## YIELD STRESS

We now present the results of the yield stress measurements performed on the same suspension. The experimental data are depicted in the left diagram in Figure 1. For the suspensions with 150 $\mu$m bubbles, the overall yield stress of the suspension is found to be the same as the yield stress of the ambient fluid.

Overvall properties of suspension of bubbles dispersed in a yield stress fluid can be estimated in the framework of the modified secant method proposed by Suquet. The key feature in these method is to write in a linear form the behavior of the suspension constituents by introducing the apparent viscosity or secant modulus. Then it is possible to replace the nonlinear upscaling problem by a linear one. Estimates for the overall yield stress of a suspension of bubbles can be computed easily when the air-fluid interfaces do not store energy. Such a condition is fulfilled when the gaz-fluid interface does not deform ($Ca = 0$) or when the surface tension is naught ($Ca \to \infty$). In these situation, the overall yield stress of a dilute suspension of bubbles reads $\tau_c(\varphi) = \tau_c(0)$ when $C_a = 0$ and $\tau_c(\varphi) = \tau_c(0) \times (1 - (4/3)/\varphi)$ when $C_a \to \infty$. These estimates are plotted in the right diagram in Figure 1. Comparison of the experimental data with the theoretical estimates shows that the capillary number of the studied suspension is close to zero in the “plastic” regime.

It remains to explain why the “plastic” capillary number is not equal to the “elastic” capillary number. The classical capillary number accurately accounts for the relative deformability of the bubble compared to that of the suspending fluid when both compenents behave elastically. This condidition is satisfied in the “elastic” regime. In the “plastic” regime, the suspending fluid yields: its secant modulus is no more constant (it is reminded that the secant modulus of a rigid plastic material reads $G_{aap} = \tau_c/\gamma$ with $\tau_c$ the yield stress and $\gamma$ the strain). As the bubble rigidity originates from air-paste surface tension, the bubbles still behave elastically in the “plastic” regime. In this situation upscaling approaches show that the overall yield stress of the suspensions is that of a dispersion of non-deformable voids ($C_a = 0$) with same size and same distribution as the bubbles dispersed in the plastic matrix. We would have obtained a completely different result for bubbles deforming in the plastic regime. Note that dilute estimates for the yield stress of a bubble suspension are decreasing functions of the bubble volume fraction, even for small bubbles.

## CONCLUSIONS

Provided that the capillary number is properly calculated, the overall properties of a a suspension of large bubbles dispersed in a yield stress fluid can be accurately estimated in the framework of an upscaling method. Nevertheless the “dilute” estimates do not allow to predict the observed trends for suspensions of small bubbles. It is believed that the increase of yield stress with bubble volume fraction observed in this situation originates from hydrodynamics interactions between bubbles. Such interactions can be taken into account using more sophisticated upscaling schemes.

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