A NUMERICAL EXPERIMENT ON BREAKUP MODES OF A LAMINAR COMPOUND JET IN A CO-FLOWING SYSTEM

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Summary We numerically investigate various breakup modes of a laminar compound jet in a co-flowing outer fluid, i.e. a co-flowing system, by a front-tracking/finite different method. Numerical results reveal that the compound jet can break up into drops in various modes: inner dripping-outer dripping, inner jetting-outer jetting and mixed dripping-jetting mode. Transition from dripping to jetting is affected by various parameters such as the Reynolds number *Re*, the Weber number *We*, the interfacial tension ratio and the velocity ratios. These modes produce various drop types: simple drops, single-core compound drop and multi-core compound drops.

INTRODUCTION

A compound jet consists of an outer jet of an intermediate (middle) fluid surrounding an inner jet of an inner fluid. It issues into an outer fluid, which is at either rest or coaxially flowing (Fig. 1a). Such a compound jet is extremely unstable, and ultimately breaks up into drops in either dripping or jetting. Dripping produces drops close to the nozzle exit whereas jetting pushes the breakup point far downstream [1],[2]. These modes are very important in compound jet applications such as encapsulation [1],[3], ink-jet printing [4]. However, there have been few studies on these modes. Theoretically, Herrada et al. [5] used a spectra-collocation method to analyze the transition from dripping to jetting in a capillary compound liquid jet moving inside a gas of negligible density and viscosity. Experimentally, Utada et al. [1] used focusing devices to form a compound jet of silicone oil–in–water inside silicone oil. They found two modes, dripping and jetting, to form compound drops that contain a single core, single–core compound drops, or a few cores, multi–core compound drops. Few numerical studies on the compound jet have been carried out. Suryo et al. [6] used the Galerkin/finite element method to analyze the finite-amplitude deformation and break-up of a compound jet in a passive gas of negligible density and viscosity. They did reveal neither dripping nor jetting since their computational domain was quite small. Zhou et al. [7] used the diffuse interface method to simulate the formation and breakup of compound jets in flow focusing devices reported in [1]. They focused only on the jetting mode, and did not produce the dripping mode. It appears that detailed numerical calculations on both these modes, dripping and jetting, of laminar compound jets in the co-flowing system are still lacking in literature. This motivates our present study on these modes. Here, we use the front-tracking method discussed in [8] to investigate various modes and transition from dripping to jetting in a laminar compound liquid jet in a co-flowing fluid in response to variation of various parameters.

PROBLEM FORMULATION AND NUMERICAL METHOD

Fig. 1a shows a typical co-flowing system of three viscous immiscible fluids. The compound jet is axisymmetric in cylindrical coordinates (*r, z*). It is injected with average velocities U_1, U_2 through a coaxial nozzle into an outer fluid that flows with an average velocity U_3 in a tube. Densities and viscosities of the three fluids, which are assumed to be immiscible, incompressible and Newtonian, are ρ_1, μ_1; ρ_2, μ_2 and ρ_3, μ_3, respectively. We assume that the inner and outer nozzles have the same wall thickness W_n of $0.2R_1$, and the same length L_n of $2R_1$. R_1 ($d_1 = 2R_1$) is the internal radius of the inner nozzle, and the internal radius of the outer nozzle is R_2 ($d_2 = 2R_2$). The governing equations are

$$\nabla \cdot \mathbf{u} = 0, \quad D\rho/Dt = D\mu/Dt = 0,$$

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla\cdot\rho\mathbf{uu} = -\nabla p + \nabla\cdot\left[\mu\left(\nabla\mathbf{u}+\nabla\mathbf{u}^T\right)\right] + \int \sigma\kappa\mathbf{n}\delta\left(\mathbf{x}-\mathbf{x}_f\right)dS + \rho\mathbf{g} = 0.$$

The boundary conditions are as follows (Fig. 1a): symmetry at $r = 0$, inflow at $z = 0$, outflow at $z = L$, and no-slip on others. Fully developed laminar flows are set at the inlet of the computational domain while the outflow condition is applied at the outlet.A set of non-dimensional parameters are the Reynolds number $Re = \rho_1 U_1 R_1/\mu_1$, the Weber number $We = \rho_1 U_1^2 R_1/\sigma_1$, the Froude number $Fr = U_1^2/gR_1$, the density ratios $\rho_{21} = \rho_2/\rho_1$ and $\rho_{31} = \rho_3/\rho_1$, the viscosity ratios $\mu_{21}=\mu_2/\mu_1$ and $\mu_{31}=\mu_3/\mu_1$, the interfacial tension ratio $\sigma_{21} = \sigma_2/\sigma_1$, the velocity ratios $U_{21}=U_2/U_1$ and $U_{31}=U_3/U_1$, and the radius ratios $R_{21}= R_2/R_1$ and $R_{31}= R_3/R_1$.

The above governing equations are solved by the front-tracking/finite difference method [8]. Detailed description as well method validation can be found in [8],[9].

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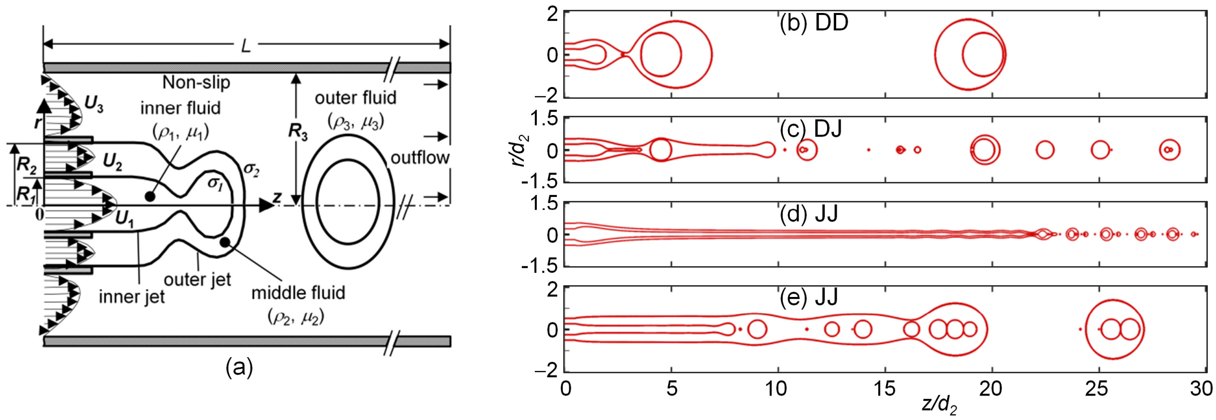

Fig. 1. (a) Sketch of a compound jet; (b) DD: inner dripping-outer dripping at U_{21}=1, U_{31}=0.4; (c) DJ: mixed dripping-jetting at U_{21}=1, U_{31}=4; (d) JJ: inner jetting-outer jetting at U_{21}=1, U_{31}=10; and (e) JJ: inner jetting-outer jetting at U_{21}=3.16, U_{31}=1. Other parameters: We =0.3, Re=5, σ_{21}=1, R_{21}=2

RESULTS

We assume that the inner and outer fluids are identical, and gravity is taken to be negligible: ρ_{31}=1, μ_{31}=1, 1/Fr=0. Moreover, ρ_{21}, μ_{21} and R_{31} are fixed at 0.6, 0.8, and 10, respectively. Figs. 1b – e show various breakup modes of the compound jet in the co-flowing system at We=0.3, Re=5,σ_{21}=1, R_{21}=2. In Fig. 1b, the inner and outer jets produce drops close to the nozzle, i.e. the dripping mode (DD). Thus, a single-core compound drop is formed. In Fig. 1c, at U_{31} = 4 the compound jet produces drops in a complex mode: the inner jet is dripping and the outer jet is jetting to form simple drops. This mixed dripping-jetting mode (DJ) forms both simple and compound drops. However, at a higher U_{31}=10 (Fig. 1d) the viscous force of the outer fluid is further increased; stretching both the inner and outer jet interfaces. Thus, a long compound filament is formed, and small compound drops with a diameter on the order of nozzle size detach at the tip of the compound jet due to capillary instability. The mode in Fig. 1d is called “inner jetting-outer jetting” (JJ). In contrast to producing small compound drops, at U_{21}=3.16 (Fig. 1e), the compound jet produce large compound drops enclosing a few inner drops, i.e. multi-core compound drops, in another inner jetting-outer jetting mode (JJ). Detailed effects of We, Re, σ_{21} and U_{31} on the drop diameter and the transition are shown in Fig. 2.

CONCLUSIONS

We have numerically revealed various modes of the compound pound jet breakup in a co-flowing system using the front-tracking technique. These modes are affected by various parameters, and produce various kinds of drops.

References

[1] Utada, A.S. *et al.*: Monodisperse Double Emulsions Generated from a Microcapillary Device. *Science* **308**:537-541, 2005.

[2] Lee, S.Y., Snider, C., Park, K., Robinson, J.P.: Compound Jet Instability in a Microchannel for Mononuclear Compound Drop Formation. *J. Micromech. Microeng.* **17**:1558-1566, 2007.

[3] Vu, T.V., Takakura, H., Wells, J.C., Minemoto, T.: Production of Hollow Spheres of Eutectic Tin-Lead Solder through a Coaxial Nozzle. *J. Solid Mech. Mater. Eng.* **4**:1530-1538, 2010.

[4] Hertz, C.H., Hermanrud, B.: A Liquid Compound Jet. *J. Fluid Mech.* **131**:271-287, 1983.

[5] Herrada, M.A., Montanero, J.M., Ferrera, C., Gañán-Calvo, A.M.: Analysis of the Dripping-jetting Transition in Compound Capillary Jets. *J. Fluid Mech.* **649**:523-536, 2010.

[6] Suryo, R., Doshi, P., Basaran, O.A.: Nonlinear Dynamics and Breakup of Compound Jets. *Phys. Fluids* **18**:082107-082107-31, 2006.

[7] Zhou, C., Yue, P., Feng, J.J.: Formation of Simple and Compound Drops in Microfluidic Devices. *Phys. Fluids* **18**:092105-092105-14, 2006.

[8] Tryggvason, G. *et al.*: A Front-Tracking Method for the Computations of Multiphase Flow. *J. Comput. Phys.* **169**:708-759, 2001.

[9] Vu, T.V., Homma, S., Wells, J.C., Takakura, H., Tryggvason, G.: Numerical Simulation of Formation and Breakup of a Three-Fluid Compound Jet. *J. Fluid. Sci. Technol.* **6**:252-263, 2011.

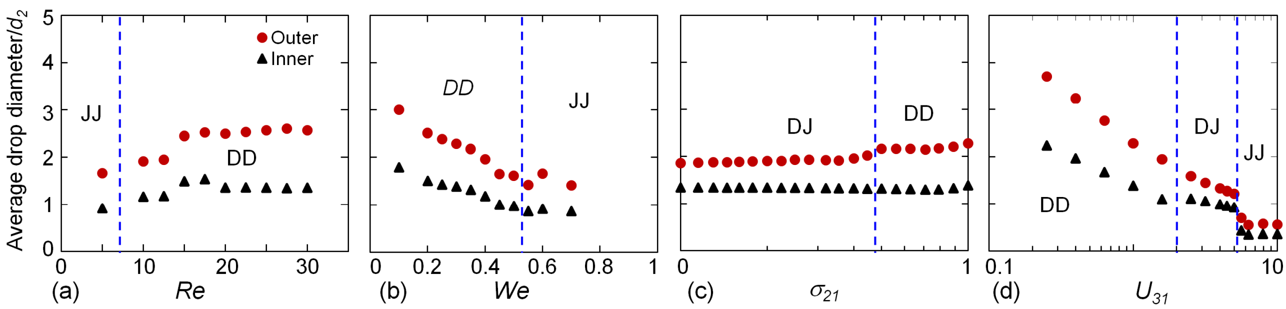

Fig. 2. Effects of various parameters on compound jet breakup modes and drop size: (a) We=0.6, σ_{21}=1, U_{21}=1, U_{31}=1, R_{21}=2; (b) Re=5,σ_{21}=1, U_{21}=1, U_{31}=1, R_{21}=2; (c) We=0.3, Re=5, U_{21}=1, U_{31}=1, R_{21}=2; (d) We=0.3, Re=5, σ_{21}=1, U_{21}=1, R_{21}=2.