VARIABLE TOPOLOGY BOUNDARY ELEMENT METHOD (VTBEM) FOR SIMULATING UNDERWATER EXPLOSION (UNDEX) BUBBLE

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Summary An underwater explosion event interacts with surrounding medium in two ways: underwater shock and underwater bubble. The present paper is only concerned with the latter. An UNDEX bubble changes its shape dramatically, expanding from a very small globe full of high-temperature high-pressure gas to a very large (several tens folds) empty cavity, followed by a violent collapse. Particularly, a collapsing bubble may lose spherical stability, even breaking into a torus (topological change). The process imposes big challenges for numerical simulations: bubble boundary changes with time (free moving boundary problem) and fluid domain experiences topological change (bubble changes into a torus). In this paper, we propose a Variable Topology Boundary Element Method (VTBEM) to handle UNDEX bubble evolution. Bubble boundary is tracked using elastic mesh technique. Through introduction of a vortex ring inside the bubble, mathematical non-uniqueness is removed. Numerical examples are provided to validate the method.

MOTIVATIONS

Bubble dynamics has been an interesting research field for one century due to its rich physical phenomena and a variety of applications. In this paper we restrict our focus on those possibly arising from Underwater Explosion (UNDEX) for their huge damaging effects on nearby structures. Two marine disasters in 2010 have close correlation with underwater bubbles: one breaking the ROK corvette Cheonan with 46 sailors died and the other sinking the Deepwater Horizon oil rig in the gulf of Mexico with 11 staff died.

Two prominent features of a bubble are well-established: bubble shape changes dramatically and a jet forms at the minimum bubble size (Fig. 1 shows two time steps). In an ordinary UNDEX event, the ratio of the maximum bubble size over the minimum bubble size can be as high as 20 or higher. The initial small bubble is full of high-temperature high-pressure gas, which drives the bubble to expand dramatically until the inside pressure is smaller than the surrounding pressure. So the pressure inside a bubble at its maximum size is very low with rarefied gas inside (cavity). Then a violent collapse follows. If the velocity on a part of the bubble boundary is much greater its counterpart at the opposite bubble boundary, the faster one catches up the lower one and joins up together (topological change), forming a jet.

If an Eulerian code is employed, it mesh density should be time-dependent, adapting to bubble size changes. Coarse mesh is employed as the bubble is large and fine mesh is used as the bubble is small. This is, however, difficult with an Eulerian code. Boundary Element Method (BEM) is thus the best fit for the problem, with mesh on the surface of the bubble co-moving with it. The high velocity of an UNDEX bubble permits neglect of viscosity and utilization of potential theory. Further revision is, however, needed so that such a BEM remains valid when topological change is present. The tricks lie in generalizing Kelvin's theorem of circulation conservation and introduction of vortex ring inside the bubble. Combining the above techniques, we propose Variable Topology Boundary Element Method (VTBEM) in this paper. Benchmarked with experimental data, the method shows good correlation.

MATHEMATICAL FORMULATION AND NUMERICAL RPOCEDURES

The fluid is assumed inviscid and incompressible, and irrotational initially. Then exists a velocity potential ϕ satisfying the Laplace equation and boundary conditions (bubble surface, free surface, sea bottom, body surface and infinity) as given in Fig. 2 and Table 1, where the gas inside the bubble is assumed ideal.

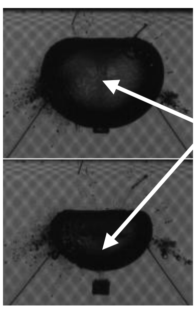

Fig 1 Bubble jet

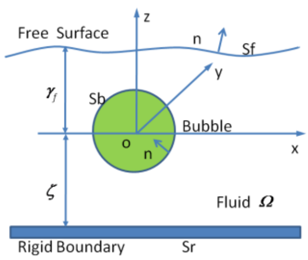

Fig. 2 Schematic of the problem

$$\nabla^2\phi = 0$$

$$\frac{D\mathbf{x}}{Dt} = \nabla\phi \quad \text{on bubble and free surfaces}$$

$$\frac{\partial\phi}{\partial n} = 0 \quad \text{on fixed rigid boundary}$$

$$\frac{D\phi}{Dt} = 1 + \frac{1}{2}|\nabla\phi|^2 - \delta^2 z - \varepsilon\left(\frac{V_0}{V}\right)^{\lambda} \text{ ,bubble surface}$$

$$\frac{D\phi}{Dt} = \frac{1}{2}|\nabla\phi|^2 - \delta^2(z - \gamma_f) \text{ ,bubble surface}$$

Table 1 Mathematical formulation

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The solution to the above problem, by virtue of Green identity, is expressed in the integral form:

$$c(\boldsymbol{P})\phi(\boldsymbol{P}) = \int_S \left(\frac{\partial \phi(\boldsymbol{Q})}{\partial n} G(\boldsymbol{P},\boldsymbol{Q}) - \phi(\boldsymbol{Q}) \frac{\partial G(\boldsymbol{P},\boldsymbol{Q})}{\partial n} \right) \mathrm{d}\boldsymbol{S}$$

Discretizing the above integral equation leads to a system of algebraic equations in the form of BEM formulation. What is special here is that the boundaries are moving, thus requiring evaluation at each time step. Triangular elements are used to approximate the bubble surface for they are able to cover a spherical surface, see Fig. 3. As the bubble grows or collapses, so does the bubble surface. The mesh composed of the triangular-elements also changes with time. As time marches, numerical error may severely deteriorate mesh quality. Then smoothing technique is employed to remove possible mesh singularity or incompatibility, see Fig. 4. As two parts of the bubble surface, which are initially far apart, meet and join to form a jet as shown in Fig. 5, a holing technique by cutting hole on the upper surface in the case of Fig. 5 is used to let water to flow through the hole. Moreover, the potential must be modified because the potentials at the upper and lower surfaces in the case of Fig. 5 are not same just before they impact, but becomes equal just after they join. The difference between the two potentials at the upper and lower surfaces before impact gives rise a finite circulation Γ, which is represented by a vortex ring in Fig. 5.

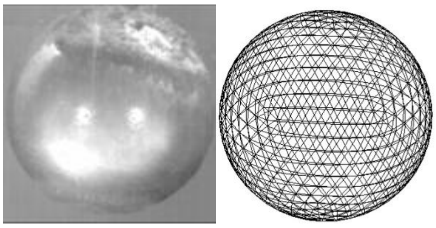

Fig. 3 Discretization of bubble surface

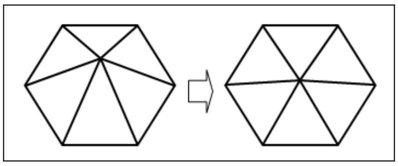

Fig. 4 Mesh smoothing

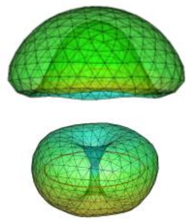

Fig. 5 Holing technique

Combining the above techniques gives the VTBEM. The code is validated with experimental data. Some of the results are shown in Fig. 6 (upper is experimental and lower is numerical). In the lower row, two observations are impressive: properly capture of dramatic bubble size change and transformation into a toroidal bubble. We also apply the method to study dynamic behaviour of a bubble in the vicinity of free surface (Fig. 7) and a ship hull as shown in Fig. 8. More details will be explained in the full paper.

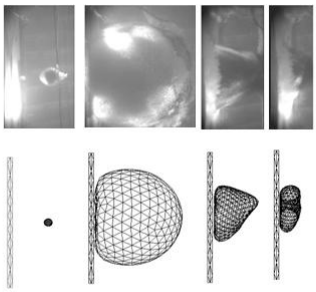

Fig. 6 Experimental validation

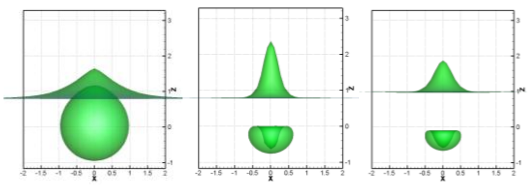

Fig. 7 Dynamic behaviour near free surface

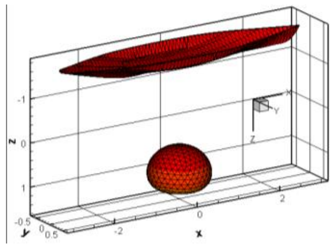

Fig. 8 bubble-hull interaction

CONCLUSIONS

In this paper the VTBEM is presented with two significant improvements: capability to follow large deformations of the bubble and capability to capture topological change. Agreement with experimental data has validated the method and code. The method is applied to simulate the dynamic behaviour of a bubble in the vicinity of free surface and a ship hull.

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