THE FARADAY INSTABILITY IN DEFORMABLE DOMAINS

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Summary Hydrodynamic instabilities are usually investigated in confined geometries where the resulting spatiotemporal pattern is constrained by the boundary conditions. Here we study the Faraday instability in domains with flexible boundaries. The instability is triggered in floating liquid lenses. An interaction of Faraday waves with the shape of the lens is observed, the radiation stress of the waves exerting a force on the surface tension held boundaries. Two regime are observed. In the first one there is a coadaptation of the wave pattern with the shape of the domain so that a steady configuration is reached. In the second one the radiations stress dominates and no steady regime is reached. The lens stretches and ultimately breaks into smaller domains that have a complex dynamics, including spontaneous propagation.

INTRODUCTION

Hydrodynamic instabilities usually appear in two types of situations corresponding to confined or opened geometries, respectively. For instance thermal buoyancy, when confined in a box, gives rise to patterns of Rayleigh-Bénard rolls adapted to the boundaries. In contrast in an open medium, an isolated source of heat generates a thermal plume in which the growing turbulent structures define the envelop of the unstable region.

In this paper we address an intermediate situation in which an instability develops inside a finite domain with flexible boundaries and we study the interplay between the pattern and its borders. We use the Faraday instability in which waves form on the surface of a fluid submitted to vertical oscillations [1]. We confine the Faraday instability in a liquid lens of low viscosity floating on on a very viscous, stable, immiscible liquid [fig.1(a)]. We have tested many pairs of liquids and finally noticed that only two archetypes of behaviours exist. In the first one there is a coevolution of the wavefield and the lens border that leads to an equilibrium shape. In the second equilibrium is not reached and the lens undergoes strong elongation and breaking [2].

FIRST ARCHETYPE OF BEHAVIOUR

The first archetype has been investigated with the pair isopropanol/fluorinated oil. Above the lens Faraday threshold, waves are created in the lens and deform its border. Above a second threshold a self-adaptation of the lens shape occurs. There is a coevolution of the wavefield and the lens border that leads to an elongated stable shape, in which the wavevector sets parallel to the long axis of the lens [fig.1(c)]. This stable shape has been explained in terms of equilibrium between the wave radiation stress exerted on the lens border and the border response due to capillarity. Radiation stress and capillary

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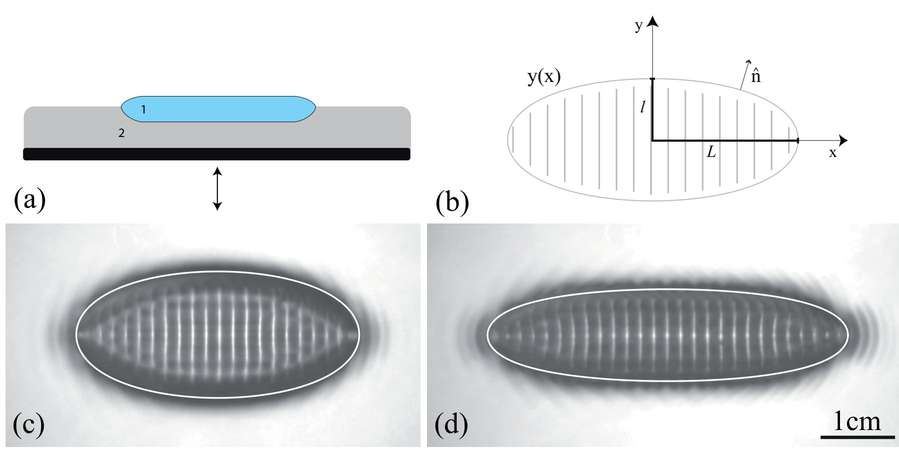
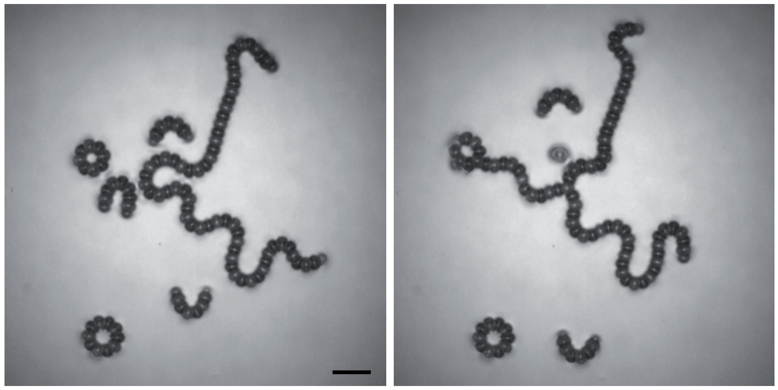


Figure 1. (a) The vibrating system made of a liquid lens floating of a bath of more viscous liquid. (b) Schematic representation of the lens in the elongated state and its horizontal contour. (c, d) Two elongations of the lens forced at $f_0 = 130$Hz and amplitude 4.05g, 5.47g respectively. The white lines are plot of the solution of eq.1.



(a) (b)

Figure 2. Two successive images of the liquid bath after the breaking of the initial lens in several parts. We notice a snake-like structure, propagating croissants and stable rings.

effects are of the same order of magnitude in this case. An accurate analysis leads to the following equation for the lens shape

$$b + a\frac{y'(x)^2}{1+y'(x)^2} = \frac{d}{dx}\frac{y'(x)}{\sqrt{1+y'(x)^2}}. \tag{1}$$

where $y(x)$ is the function describing the lens contour in the horizontal plane [fig.1(b)], and a is a parameter that takes into account both radiation stress and capillarity. This equation has an exact solution that we have superposed to experimental figures. The agreement is excellent for many elongations of the lens [fig.1(c, d)]. This analytical model also allows calculation of the lens aspect ratio, that is the ratio of its width over its length. Even in this case the agreement of the experimental data with the theoretical results is excellent.

SECOND ARCHETYPE OF BEHAVIOUR

The second archetype has been investigated for the pair ethanol/silicon oil. The silicon oil wets the ethanol and the surface tension between the two liquids is very low, about one order of magnitude than that of the first archetype. As a consequence the lens shape is easily deformable. Once the Faraday instability is triggered in it, the lens elongates rapidly decreasing its thickness. The radiation stress is now about 20 times the effect of capillarity. The lens is stretched and assumes a snake-like structure that is not stable, but continuously evolving. An equilibrium configuration is not achieved in this case. Only a dynamical evolution of the system can lead to equilibrium shapes. The snake-like structures moves and curves continuously on the bath. This can lead to a breaking of the structure in two separated branches that continuously evolve. After several breaking small structure are generated on the bath surface and they have a complex dynamics, so that the surface looks like a physical version of the Conway's game of life [fig.**??**]. However, at least three equilibrium shapes are achieved for small structures: simple elongated one, croissant drifting with constant velocity and ring. The dynamics of these structures is a current object of study.

CONCLUSIONS

We have investigated the Faraday instability in floating liquid lenses that behave as deformable domains. Faraday waves interact with the lens border leading to equilibrium or out-of-equilibrium behaviours of the lens shape. In a first archetype of behaviour a steady shape is reached as a consequence of the equilibrium between the wave radiation stress and the capillary response of the border. A theoretical analysis has lead to prediction of the lens shape and its aspect ratio, showing an excellent agreement with the experimental results. In the second archetype the wave radiation stress exceeds the response of the border and equilibrium is reached only after fragmentation of the initial lens. This second behaviour required further theoretical investigations.

References

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[2] Pucci G., Fort E., Ben Amar M. and Couder Y.: Mutual adaptation of a Faraday instability pattern with its flexible boundaries in floating fluid drops *Phys. Rev. Lett.*, **106**, 2011.