INCREASE OF THE MEAN VELOCITY OF A BUBBLE RISE IN A VISCOPLASTIC FLUID INDUCED BY AN OSCILLATING PRESSURE FIELD

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Summary It has been documented experimentally and theoretically that increasing the yield stress of a viscoplastic material up to a critical value leads to entrapment of a deformable bubble in it. This phenomenon plays a crucial role in producing (un)desirable products in a number of industrial processes in the food, cosmetics and building industries and in reducing the transfer of oxygen in fermentation processes. Here we examine the conditions under which an applied oscillatory pressure field in the ambient material leads to increasing the bubble mean rise-velocity or induces mobilization of an entrapped bubble. We have found that this velocity increase is maximized when the forcing frequency of the pressure oscillations is close to the eigenfrequency for bubble volume-oscillations in the viscoplastic material. Other parameters that affect it are the bubble deformability and size and the material viscosity and shear-thinning measured by the Bond and Archimedes numbers and the shear-thinning exponent.

INTRODUCTION

Bubble motion and entrapment in viscoplastic materials has many applications in industry [1-5]. Some examples include (i) the aeration of wastes or enhancing oxygen transfer in fermentation processes, (ii) the bubble entrapment in structural materials (i.e. cement, adhesives) which affects their mechanical properties, (iii) bubble formation and evolution during drilling of mud (which is of great importance in offshore oilfield drilling) and (iv) in producing desirable foods (e.g. chocolate) and cosmetics. Typical viscoplastic materials are high concentration suspensions of solid particles or macromolecules. One such material that is often used in experiments because it is transparent is Carbopol which is a poly-acrylic acid suitably neutralized. Carbopol exhibits a yield stress, because of the increased links between chains in the case of low rate-of-strain as well as the shear reduction of viscosity due to gradual destruction of entanglements when the applied rate of strain is increased [4, 5]. A bubble generated in a viscoplastic material will rise with a constant velocity and deform, depending on its size, the density difference between the material and the gas in the bubble, the material yield stress, viscosity and shear thinning exponent and the interfacial tension. These properties are grouped together in the Bingham, Archimedes and Bond numbers. Unyielded material arises at the bubble equator and far from the bubble, because the second invariant of the stress tensor necessarily falls below the yield stress there [2]. Moreover as the yield stress of the material increases the two unyielded areas expand and eventually merge leading to bubble entrapment. This occurs at a critical Bingham number the value of which depends on the Bond and Archimedes numbers and is something that crucially affects the products and processes mentioned above and it can be desirable or undesirable. When an oscillatory pressure field is applied on the ambient fluid it will force the bubble to undergo volume oscillations, in addition to its rise velocity. This additional motion increases the rate-of-strain tensor and thus reduces the plastic material viscosity, increasing the mean rise-velocity of the bubble. Here we examine the conditions under which the applied oscillatory pressure field in the ambient fluid will maximize the bubble rise velocity.

GOVERNING EQUATIONS

The ambient fluid is subjected to an acoustic pressure given by $P_\infty = P_{ls} + \varepsilon \sin(\omega_f t)$, where P_∞ is the pressure in the fluid far from the bubble, P_{ls} the static pressure (and that at t = 0), ε the amplitude of the disturbance and ω_f the forcing frequency. The flow of the liquid is governed by the momentum and mass conservation equations. For the constitutive equation that describes the rheology of the fluid, the continuous equation proposed by Papanastasiou [6] is used. This equation in dimensionless form is $\underline{\underline{\tau}} = Bn(1-e^{-N\gamma})/\gamma)\underline{\underline{\gamma}}$, where γ is the 2nd invariant of the rate-of-strain tensor, the Bingham number is defined as $Bn = \tau_y/\rho g R_b$, τ_y denotes the yield stress, and N is the dimensionless stress growth exponent $N= n\rho g R_b/\mu_0$, where n is a model parameter. We have extended this model to account for shear thinning fluids. The pressure inside the bubble varies adiabatically depending on its instantaneous volume and is calculated as part of the solution through $P_g = P_{gs}(V_s/V)^k$ where k is the polytropic exponent, V and V_s are the volume of the bubble at the given time and at t = 0 respectively, P_g and P_{gs} the corresponding pressures of the gas. The oscillatory pressure field induces volume oscillations on the bubble. Their eigenfrequency depends on the Archimedes number ($Ar= \rho^2 g R_b^3/\mu_0^2$), the Bond number ($Bo= \rho g R_b^2/\sigma$ where σ is the surface tension), the Bingham number, the initial pressure of the liquid far from the bubble, the polytropic exponent and the radius of the bubble.

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NUMERICAL SOLUTION

The above set of equations has been solved using the mixed finite element method to discretize the velocity and pressure fields, combined with an advanced elliptic grid generator for the initial construction and subsequent deformation of the mesh points in the liquid domain. The grid generation scheme that has been employed consists of a system of quasi- elliptic partial differential equations, capable of generating a boundary fitted discretization of the deforming domain occupied by the liquid; as described in Dimakopoulos & Tsamopoulos [7]. This scheme is capable to relocate the grid points in response to the bubble surface evolution and the resulting liquid volume distortion [7] and is easily implemented to domains that require domain decomposition and multiple mappings [2, 7, 8].

RESULTS

In order to study the influence of the properties of the liquid, the bubble size and the applied pressure to the rising velocity of the bubble, we performed a parametric study with respect to the values of the previously mentioned dimensionless numbers as well as the amplitude and the frequency (expressed in terms of the Strouhal number) of the applied pressure. Our results so far show that the bubble rise velocity undergoes an oscillation with a phase difference form the pressure oscillation. More importantly, its mean value increases over time tending to an asymptotic value as depicted in figure 1 for Ar=10, and $\varepsilon = 4$, and in figure 2 for Ar = 5, and $\varepsilon = 4$. In both cases the forcing frequency is equal to the eigenfrequency for bubble volume oscillations and the mean bubble velocity increases by about 50% with respect to its value at a constant ambient pressure. When the forcing frequency departs from the eigenfrequency this increase is diminished until it becomes insignificant.

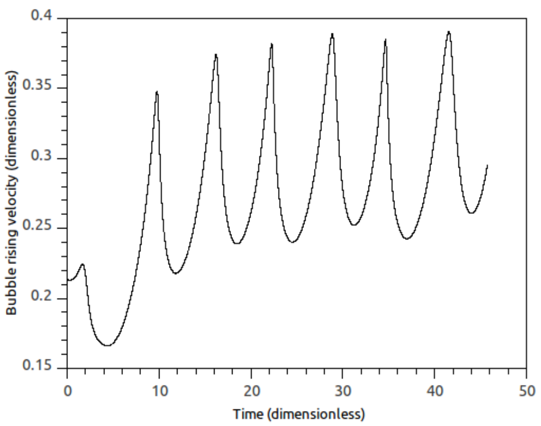

Figure 1

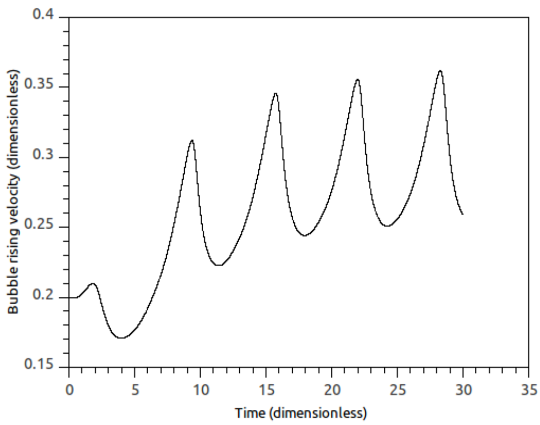

Figure 2

In addition, we have carried out detailed calculations to verify that using the continuous Papanastasiou model produces results similar to those one would obtain with the discontinuous Bingham or Herschel-Bulkley fluid models [5]. To this end we have solved the latter models using the augmented Lagragian approach. The main conclusion from this study is that as long as the exponent in the Papanastasiou model is kept large enough (its exact value depends on the specific problem studied and the size of the 2nd invariant of the rate-of-strain tensor) the results of the two models are in agreement. For example, when the level of viscoplasticity becomes large enough to nearly cause bubble entrapment, the normalization exponent needs to obtain extremely large values (of the order of 10^6) to approximate accurately the distinct regions (yielded and unyielded) of the material.

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