PREDICTION OF BUBBLE SOUND EMISSIONS USING COUPLED OSCILLATORS

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Summary Bubble sounds potentially offer valuable data on many systems in nature and industry. In reality, multiple bubbles are always present, confusing measurements. A coupled-oscillator model shows that the pulse of sound typically measured represents beats between eigenmodes of a multi-bubble system. Using only the first two eigenmodes of the system, the base and beat frequencies of experimental data can be predicted, permitting the size and spacing of bubbles to be related to the pulse frequencies alone.

INTRODUCTION

Bubbles are fundamental to processes ranging from chemical engineering and metallurgy to breaking ocean waves and erupting volcanoes (see [1] and references therein). Knowledge of the average size and relative spacing of the bubbles formed would give the interfacial surface area generated, yielding vital information on the rate at which gases are dissolved into liquid. In just one example, it is thought that up to half of global CO_2 production may be absorbed by the oceans as waves break, but the true global rate is unknown - and hitherto unmeasurable [1]. The sounds naturally emitted by bubbles on formation are a potential source of such data. Underwater sound is very easily measured.

For the small-amplitude oscillations creating most natural and process-engineering sounds, the spherically-symmetric momentum and continuity equations in the liquid give the natural acoustic frequency of an isolated bubble, f_0 (in Hz), as

$$2\pi f_0 = \omega_0 = \sqrt{\frac{3\kappa P_0}{\rho}\frac{1}{R_0}}, \tag{1}$$

[2] where κ is the polytropic exponent of gas inside the bubble, P_0 is the absolute liquid pressure and ρ is the liquid density; these are effectively constant for natural and engineering bubbles, so f_0 is inversely proportional to the equilibrium radius R_0. In principle, therefore, R_0 can be determined from the easily-measured f_0 [3].

However, attempts to apply (1) to extract data from complex, turbulent bubbly flows have been semi-empirical, owing to complexities introduced by the presence of other bubbles [3, 4]. Because the other bubbles may be perturbed by differing amplitudes, in practice, an arbitrary parameter is needed to generate bubble-size spectra. This arbitrary parameter either directly or indirectly involves an assumption on the amplitude of the sound created on bubble formation [5].

THEORY

Bubbles as acoustically coupled oscillators

The multiplicity of bubbles found in practical flows create theoretical issues including the breaking of spherical symmetry and multiple scattering (infinite re-reflexions of sound between interacting bubbles); issues usually circumvented using the 'self-consistent' methodology [6]. For linear dynamics, the resulting multiple-bubble equations are

$$\ddot{x}_i + b_i\dot{x}_i + \omega_{0i}^2 x_i = -\sum_{j=1, j\neq i}^{N} \frac{R_{0j}}{s_{ji}}\ddot{x}_j, \tag{2}$$

[7] in which $x_i(t)$ represents the instantaneous perturbation in the radius of the ith bubble, its damping constant is b_i and ω_{0i} is given by (1). The sum on the right-hand side of (2) represents the acoustic interaction with other bubbles j, with centres spaced s_{ji} away. Numerical solutions of systems like (2) can predict multiple-bubble sound emissions [8].

Two-mode model of bubble sound emission

Now consider only the interaction between a bubble and its nearest neighbour. Owing to the R_{0j}/s_{ji} factor in (2), this interaction is likely to dominate. This is relevant in many industrial cases in which a bubble is formed from an injection point, causing it to oscillate acoustically, with a virtually identical previously-formed bubble a short distance away [3]. Then (2) has an analytic solution: two eigenmodes, denoted '+' and '-', and for equal-sized bubbles, the eigenvector, natural frequency and damping constant are respectively given by

$$(1,1), \quad \omega_+ = \frac{1}{\sqrt{1+R_0/s}}\,\omega_0, \quad b_+ = \frac{b}{1+R_0/s}, \tag{3}$$

for the low-frequency or 'symmetric' mode, in which bubbles oscillate in phase, and

$$(1,-1), \quad \omega_- = \frac{1}{\sqrt{1-R_0/s}}\,\omega_0, \quad b_- = \frac{b}{1-R_0/s}, \tag{4}$$

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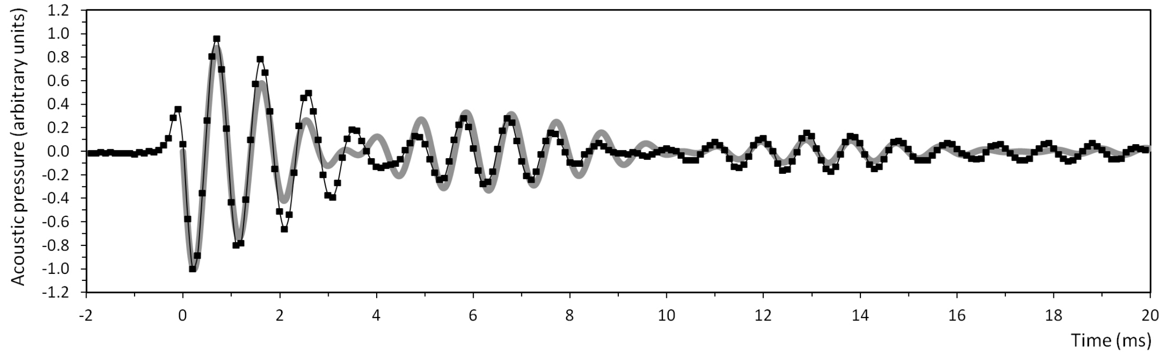

Figure 1. Pulse of sound produced by a bubble at the bottom of a chain of about 12 bubbles. Thick grey line: analytic two-mode model (equation 5). Symbols: experimental data from [3]. Both model and data were normalised so their maximum values are unity.

for the high-frequency or 'antisymmetric' mode, in which bubbles oscillate in antiphase [7]. Since only the newly-formed bubble is initially perturbed while the older bubble is acoustically at rest, this initial condition is represented by an equal sum of the two eigenmodes. The acoustic pressure that would be measured in practice, p, is directly proportional to the fluid displacement velocity, so the oscillation resulting from the two equally-perturbed modes is given by

$$p = A \sin(\omega_- t)\ \mathrm{e}^{-b_- t} + A \sin(\omega_+ t)\ \mathrm{e}^{-b_+ t}, \tag{5}$$

where the amplitude A is determined by the nature of the perturbation given to the newly-formed bubble. A beating between the two modes can be expected, with a base frequency $\frac{1}{2}(\omega_- + \omega_+)$ and beat frequency $\frac{1}{2}(\omega_- - \omega_+)$.

COMPARISON WITH EXPERIMENT

The equally-perturbed two-mode model (5) and experimental data points from [3] are shown in Figure 1. The experimental bubble radius was about $R_0 = 3$ mm, for which standard data [2] give $b/\omega_0 = 0.03$, and the experimental interbubble spacing was $s_{ij} = 23$ mm. A total of about 12 bubbles were present including the newly-formed bubble. The beat period of the combined oscillation is very well predicted, as is the base frequency. Differences between the analytic prediction and data are noticeable close to the minima of the beat envelope, where weaker modes from the interaction with other bubbles may have a chance to influence the signal. The model prediction also dies off slightly faster than the experimental data in the last few milliseconds. Most significantly, however, the apparently complex behaviour of the pulse is clearly predicted by only two modes, despite the potential influence of about 10 other modes. Variations in R_0 and s_{ij} beyond the approximately 10% experimental error produced poor comparisons, implying that the model could be a sensitive determinant of R_0 and the spacing s_{ij}.

CONCLUSIONS

Numerically evaluated coupled-oscillator models can predict bubble sound emissions, but an analytic model is needed to efficiently extract bubble data of practical value from experimental sounds. An equally-perturbed two-mode model of the sound emitted by a newly-formed bubble accurately predicted the apparently complex sound pulse measured when multiple bubbles are present. Only two parameters are required for the model, the bubble size and the interbubble spacing; and two variables can be measured, the base and beat frequencies of the pulse. Thus, no arbitrary parameters are necessary. For a multi-bubble system in which bubbles are generated by a reproducible process so that bubbles are approximately monodispersed, the bubble size and interbubble spacing can be determined by solely by measurements of the frequencies of sound naturally emitted. In principle, the interfacial area generated can thus be measured acoustically.

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