INTERACTION OF RISING BUBBLE AND PARTICLE IN OSCILLATING LIQUID

Dmitriy Lyubimov[*a)], Lyudmila Klimenko[**], Tatyana Lyubimova[**] & Lev Filippov[***]
[*] *Perm State Research University, Perm, 614990, Russia*
[**] *Institute of Continuous Media Mechanics UB RAS, Perm, 614013, Russia*
[***] *Institut National Polytechnique de Lorraine, Vandoeuvre-lès-Nancy, F-54501, France*

Summary The interaction of rising bubble and small rigid particle in a liquid subjected to vibrations is studied. It is assumed that vibration frequency is such small that the liquid can be considered as incompressible. On the other hand Stokes length is assumed to be small in comparison with all characteristic length scales. In these conditions the bubble performs translational oscillations and is subjected to the periodical variations of radius. Average force acting on the particle is calculated. It is shown that main contribution into the force is made by interference of external field and field caused by monopole mode of bubble oscillations and besides the interaction force is the attraction of particle to the bubble. It is found that the vibrations lead to considerable increase of the efficiency of particle capture by the bubble. The possibility of application of this effect to the intensification of flotation process is discussed.

Behaviour of single heavy rigid particle near a gaseous bubble in incompressible liquid subjected to the gravity and external vibrational field is studied. Vibrations are translational and harmonic, with amplitude b and frequency ω. Vibration frequency is assumed to be such high that the thickness of the boundary layer inside of which the viscous dissipation takes place is much smaller than both the bubble and particle radii. This allow to use the model of the non-viscous fluid for the description of pulsational flow around the bubble and particle.

The total force acting to the particle consists of the average force created by the vibrational flow, the Stokes force, the gravity and buoyancy forces.

Momentum equation for particle motion written in the reference frame of the bubble taking into account the particle inertia and added mass and averaged over the time scale large in comparison with the vibration period has the form:

$$\left(\rho_s+\frac{1}{2}\rho\right)\frac{d^2\vec{r}}{dt^2}=\frac{3}{4}\rho\frac{\rho_s-\rho}{\rho_s+\frac{1}{2}\rho}\nabla\overline{V^2}-\frac{9}{2R_p{}^2}\eta\left(\frac{d\vec{r}}{dt}-\vec{U}\right)+\left(\rho_s-\rho\right)\vec{g}\,,$$

where ρ_S is the particle density, ρ,η are the liquid density and dynamic viscosity, R_p is the particle radius, $\vec{U}$ is the liquid velocity in the absence of particle, $\vec{V}$ is the velocity of the pulsational flow near the particle.

Pulsational velocity field includes external field and scattered fields of monopole and translational motions:

$$\vec{V}=\vec{\mathrm{V}}_{vib}+\vec{\mathrm{v}}_r+\vec{\mathrm{v}}_d=\vec{\mathrm{V}}_{vib}+\left(\vec{\mathrm{V}}_{vib}\cdot\vec{r}_b\right)\frac{\omega^2R_b}{\left(\omega^2-\omega_0^2\right)}\frac{\vec{r}}{r^3}-\vec{\mathrm{V}}_{vib}\frac{R_b{}^3}{r^3}+\left(\vec{\mathrm{V}}_{vib}\cdot\vec{r}\right)\frac{3R_b{}^3}{r^4}\frac{\vec{r}}{r}$$

Here $\vec{\mathrm{V}}_{vib}=b\omega\vec{j}\cos\omega t$ is the velocity of pulsational flow far from the bubble, $\omega_0=c/R_b\sqrt{3\rho_g/\rho}$ is the eigen frequency of radial oscillation of the bubble, R_b is the bubble radius, $\vec{r}_b$ are coordinates of the bubble with respect to the pressure node, ρ_g is the gas density in the bubble, c is the sound velocity in gas.

In Fig. 1 the trajectories of particle motion in vertical plane passing through the vibration direction are presented. In the absence of vibrations, at the distance from the bubble or the order of its diameter, the influence of bubble on particle motion is weak, the particle trajectories are nearly vertical. In the presence of vibrations the deviation of particle trajectories from vertical ones takes place and at some values of the vibration parameters the particle is attracted to the bubble surface.

Complete description of the vibration influence of the collision efficiency can be obtained by calculating the particle capture cross-section, i.e. the areas in the horizontal plane located far enough from the bubble, approaching to which the particles will be attracted to the bubble surface (Fig.2). In the absence of vibrations and particle inertia the particle capture cross-section is axisymmetric. Even weak vibrations destroy axial symmetry and lead to considerable (in several

[a)] Corresponding author. Email: lyubimov@psu.ru

times) increase of the particle capture cross-section which should result in the growth of the total efficiency of the flotation process.

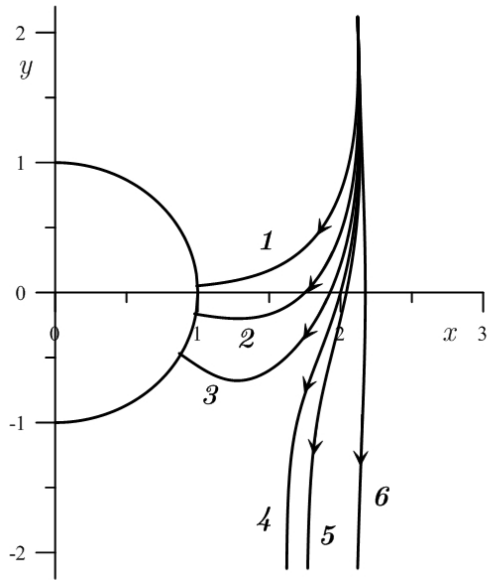

Fig.1. Trajectories of particle motion. 1 – in the absence of external flow, 2 – $b\omega = 1.88$ м/с, 3 – $b\omega = 0.94$ м/с, 4 – $b\omega = 0.62$ м/с, 5 – $b\omega = 0.47$ м/с, 6 – in the absence of vibrations; $\rho_s / \rho = 2$, vibrations are parallel to x-axis, gravity force – to y-axis

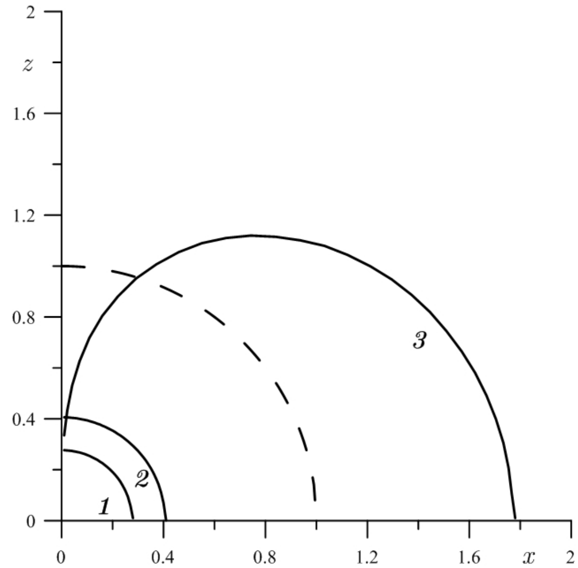

Fig.2. Particle capture cross-section: domain 1 – in the absence of vibrations and inertia, domain 2 – in the absence of vibrations but accounting for the inertia, domain 3 – in the presence of vibrations and inertia ($b\omega = 0.62$ m/s). Dashed line – projection of the bubble cross-section

ACKNOWLEDGMENTS

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