ELECTRIC FIELD INDUCED DROP-DROP INTERACTIONS

Gaurav Tomar*[a)]
*Department of Mechanical Engineering, Indian Institute of Science Bangalore, India - 560012

Summary We present here numerical simulations of drop-drop interactions in the presence of electric field. Coalescence or separation of drops occurs due to the hydrodynamic forces generated by the electric field induced flows. The nature of the interaction depends upon the ratios of the electric properties of the drops and the ambient fluid.

INTRODUCTION

Drops (bubbles) in emulsions (foams) can be stabilized or coalesced using an electric field. A surface force acts at the drop interface due to an applied external electric field. Ratios of the electric properties of the drop and the ambient fluid defines the nature of drop deformation (oblate or prolate) and interactions between them. Interaction between drop pairs was analyzed analytically by Sozou [1] and numerically by Baygents et al.[2] under the Stokes flow limit. We present here numerical simulations at finite Reynolds numbers using a Coupled Level Set and Volume-Of-Fluid (CLSVOF) algorithm [3] to study the drop-drop interactions in the presence of an externally applied electric field.

FORMULATION

We model the drops and the ambient fluid using Taylor's leaky-dielectric model [4]. The governing equation for the electric field is given by,

$$\nabla \cdot \mathbf{J} = \nabla \cdot (\sigma \mathbf{E}) = 0. \tag{1}$$

Here, $\mathbf{J}$ is the electric current density in the medium, σ is the electric conductivity and $\mathbf{E}$ is the electric field strength . The electromagnetic effects can be ignored ($\nabla \times \mathbf{E} = 0$, $\mathbf{E} = -\nabla\psi$ where ψ is the electric potential) since the corresponding characteristic time scale is much shorter as compared to the charge relaxation time $\epsilon\epsilon_0/\sigma$. Also, assuming the charge relaxation time to be much faster compared to the fluid time scale implies that the charge accumulation occurs only at the interface ($\nabla \cdot \mathbf{E} = 0$ in the drops and the ambient fluid).

The governing equations for the fluid motion are the Navier-Stokes equations, which are modified to account for the electric field force ($\mathbf{f}_v^E$),

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\left[\mu(\nabla\mathbf{v} + \nabla\mathbf{v}^T)\right] + \mathbf{f}_v^\gamma + \mathbf{f}_v^E. \tag{2}$$

Here, $\mathbf{v}$ is the fluid velocity, p is the pressure, ρ is the density and μ is the viscosity of the fluid. The fluid is considered to be incompressible, $\nabla \cdot \mathbf{v} = 0$. To simulate two-phase electrohydrodynamic flows we use a CLSVOF algorithm [3]. In the CLSVOF framework, the normal stress and tangential stress balance boundary conditions at the fluid-fluid interface are implicitly imposed. The surface tension force and the electric field force which act only at the interface for homogeneous mediums are modeled as volumetric forces. The volumetric surface tension force can be written as $\mathbf{f}_v^\gamma = \gamma\sigma\mathbf{n}\delta_S$, where δ_S is the surface Dirac delta function. The Dirac delta function is smoothed over a transition region (3 grid cells around the interface in the present study) in the CLSVOF scheme. The details of the algorithm used here are given in Tomar et al.[3]. The volumetric electric field force is given by [3],

$$\mathbf{f}_v^E = -\frac{1}{2}\epsilon_0 E^2\nabla\epsilon + q_v\mathbf{E} = \frac{\epsilon_0}{2}\left[\nabla\left(\frac{\epsilon}{\sigma^2}\right)(\sigma\mathbf{E}\cdot\mathbf{n})^2 - (\mathbf{E}\cdot\mathbf{t})^2\nabla\epsilon\right] + \epsilon_0\left[(\sigma\mathbf{E})\cdot\nabla\left(\frac{\epsilon}{\sigma}\right)\right](\mathbf{E}\cdot\mathbf{t})\,\mathbf{t}. \tag{3}$$

Here $\mathbf{t}$ is the tangent vector at the fluid-fluid interface, q_v is the free charge density ($q_v = \nabla\cdot(\epsilon\mathbf{E})$) and ϵ is the relative permittivity of the medium.

RESULTS AND DISCUSSIONS

We perform axi-symmetric simulations of interactions between two drops alligned along the electric field. Depending upon the ratios of relative permittivities and electric conductivities of the drops and the ambient fluid, the drops may deform oblately or prolately[4]. In addition, depending upon these ratios, they may approach each other and coalesce or repel each other[1, 2]. The capillary number is defined as $Ca = \mu_a R/\gamma$, where μ_a is the viscosity of the ambient fluid and R is the radius of the initial spherical drops. An electric capillary number can be defined as the ratio of the electric field forces to the surface tension, $Ca_E = \epsilon_a\epsilon_0 E_\infty^2 R/\gamma$. Reynolds number can be defined based on typical velocities in electrohydrodynamic viscous flows ($u_{EH} = \mathbb{O}(\epsilon_a\epsilon_0 E_\infty^2 R)/\mu_a$), $\mathbb{R} = \rho_a\epsilon_a\epsilon_0 E_\infty^2 R^2/\mu_a^2$. Figure 1(a) shows two drops approaching each other. The permittivity and conductivity ratios of the drops (represented by subscript d) and the ambient fluid (denoted by subscript a) are $\epsilon_d/\epsilon_a = 8$ and $\sigma_d/\sigma_a = 6$, respectively. The non-dimensional parameters for the simulation are: $Ca = 0.01$, $Ca_e = 1.5$ and $\mathbb{R} = 15$. The drops approach each other and coalesce. On their approach the drops taper at their approaching ends because of the non–uniform electric field (see electric field streamlines in the left half of the Figure 1a) in the region between the drops. Tangential electric stresses at the interface causes a tangential fluid acceleration leading to circulations in the drops and the ambient fluid (see flow streamlines in the right half of the figure 1a). The attraction between the drops is the effect of the fluid motion around the drops [2]. Figure 1b shows interaction

[a)] Corresponding author. E-mail: gtom@mecheng.iisc.ernet.in.

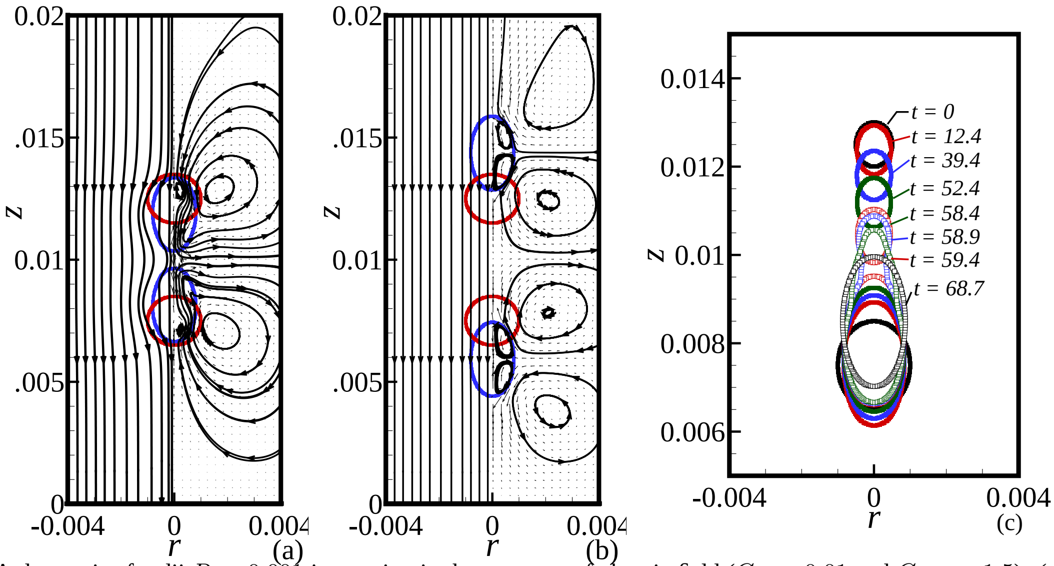

Figure 1. A drop pair of radii $R = 0.001$ interacting in the presence of electric field ($Ca = 0.01$ and $Ca_E = 1.5$). (a)$\epsilon_d/\epsilon_a = 8$, $\sigma_d/\sigma_a = 6$ (b)$\epsilon_d/\epsilon_a = 0.2$, $\sigma_d/\sigma_a = 1.04$ (c)$\epsilon_d/\epsilon_a = 8$, $\sigma_d/\sigma_a = 6$ with radius of one drop being $R/2$.

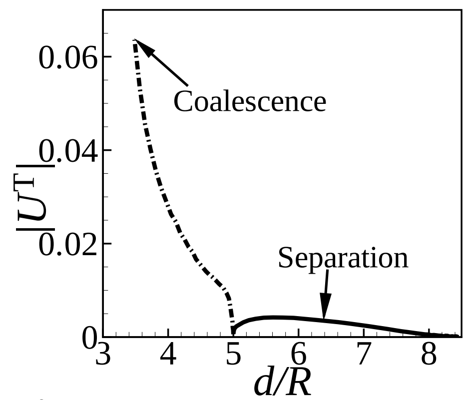

Figure 2. Relative velocity of the two drops during coalescence (Figure 1a) and separation (Figure 1b).

between two drops with $\epsilon_d/\epsilon_a = 0.2$ and $\sigma_d/\sigma_a = 1.04$ for the same values of Ca, Ca_E and $\mathbb{R}$. The drops deform prolately and move away from each other. The electric-field streamlines (left half of the figure) are uniform because the conductivity ratio is close to one. Thus, the drops do not interact electrically. The tangential electric forces are in the opposite direction as compared to the previous case (Figure 1a). Therefore, drops interact only hydrodynamically and move away from each other. Figure 1(c) shows two drops of unequal sizes in the presence of electric field at different (non-dimensional) time instances ($t = t^*/T$ where $T = R/u_{EH} = 0.007$ is the electrohydrodynamic time scale). The smaller drop experiences greater acceleration and traverses towards the larger drop and coalesces. The initial deformation into prolate spheroids occurs over a very short time interval ($t = 0 - 12.4$) and as they approach, the smaller drop becomes slightly more prolate and coalesces with the bigger droplet at $t = 58.9$ to form a single prolately deformed droplet ($t = 68.7$). Figure 2 shows relative velocities of the drops at different separation distances (d). The relative velocity (U^T) of coalescing drops initially increases linearly and expedites nonlinearly towards coalescence. Whereas, relative velocity for separating drops initially increases and then decreases to zero as drops move farther.

CONCLUSIONS

Drops in a leaky-dielectric emulsion interact with each other in the presence of an externally applied electric field. The nature of the interaction is hydrodynamic where the flow is governed by the tangential electric stress at the drop interface. Drops may coalesce or repel depending upon the strength of the circulation generated between the drops in the ambient fluid. The results presented here are for finite Reynolds number in contrast to those of Baygents et al.[2] which were performed under the Stokes flow assumption using boundary element method.

References

[1] Sozou C: Electrohydrodynamics of a pair of liquid drops. *J. Fluid Mech* **91**:541-546, 1975.
[2] Baygents J.C., Rivette N.J., Stone H.A.: Electrohydrodynamic deformation and interaction of drop pairs.*J. Fluid Mech* **368**:359-375, 1998.
[3] Tomar G., Gerlach D., Biswas G., Alleborn N., Sharma A., Durst F., Welch S.W.J., Delgado A.: Two-phase electrohydrodynamic simulations using a volume-of-fluid approach.*J. Comput. Phys.* **227**:1267-1285, 2007.
[4] Taylor G.I.: Studies in electrohydrodynamics: I. The circulation produced in a drop by an electric field. *Proc. R. Soc. Lond. A* **291**:159-166, 1966.