INSTABILITIES AND ENCAPSULATION IN A GAS-CORED COMPOUND LIQUID JET

Takao Yoshinaga*
Department of Mechanical Science, Osaka University, Osaka 560-8531, Japan

Summary Instabilities and encapsulation phenomena are analytically investigated in a gas-cored compound liquid jet. Under the long wave approximation, a set of reduced nonlinear equations is derived for the core and annular phases. Breakup of the jet is numerically examined in the equations when the jet is semi-infinite and sinusoidal disturbances are fed at the end of the jet. It is found that there exit the most unstable frequencies of disturbances giving the shortest breakup time, which are different from the linear analysis. In particular, it is shown for small Weber numbers that the core phase is encapsulated through the pinching of the jet and the resulting capsule formation periods and sizes can be determined by using these most unstable frequencies, which well agree with the results in the previous experiment and the existing phenomenological model.

INTRODUCTION

A gas or liquid-cored annular jet, which is called a compound jet, is of great importance in various engineering and industrial applications such as coating, encapsulation and atomization techniques. It has been experimentally shown that the jet breaks up by sealing-off or ballooning of the annular phase and finally the core phase is encapsulated by the annular one [1, 2]. In particular, for a gas-cored liquid jet emanating from an annular nozzle, Kendall [2] observed that a train of liquid capsules is naturally produced and its formation frequencies depend upon velocity ratios of the core to the annular. Although such frequencies were partially predicted by Lee and Wang [3, 4] in their phenomenological model, their analysis was based on a thin annular liquid sheet without a core flow under a temporally variable gas pressure.
However, it is expected that the encapsulation is affected large by rapid changes of the local pressure and flow velocity at a bottle neck of the core phase near the pinching. Therefore, the motion of the core fluid, even a gas, is essential not only in the breakup but also at the capsule formation. In this paper, such a capsule formation process is considered from the view point of the nonlinear instability.

FORMULATION

Figure 1 shows the schematic of the jet in the (z, r) axisymmetric coordinate system. Noting that the subscript $j = 1$ is refereed to as the core phase or the inner surface and $j = 2$ as the annular phase or the outer surface, the velocity vectors are denoted by $\boldsymbol{u}_j$ whose components are (u_j, v_j), while the densities by ρ_j, the surface tensions on the surfaces by σ_j and the pressures by p_j. The surfaces are specified at the radii $r = h_j$, while the pressure of the ambient gas p_3 is constant and the gas density is ignored. For convenience of the later analysis, the thickness of the annular phase $b\,(= h_2 - h_1)$ and the radius of the mid-plane $R\,(= (h_2 + h_1)/2)$ are introduced. In the analysis, we assume that the core and annular liquids are inviscid and incompressible, and the gravitational force is negligible.

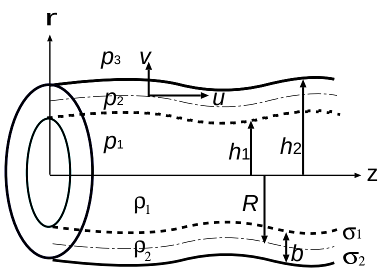

Figure 1. Schematic of the compound liquid jet.

The basic equations are given by the continuity and momentum equations for the core phase $(0 < r < h_1)$ and the annular phase $(h_1 < r < h_2)$, while the boundary conditions are given by the kinematic and dynamical conditions on $r = h_1$ and h_2. These basic equations and the boundary conditions can be simplified by using the long wave approximation, in which the dependent variables in the core phase are expanded in terms of r^2, while those in the annular phase in terms of $r - R$. Using these expansions, we finally obtain the following reduced equations in the lowest order of the approximation [5]:

$$\partial b/\partial t = -\partial(bu_2)/\partial z - bv_2/R, \quad \partial R/\partial t = v_2 - u_2 \partial R/\partial z, \quad \partial u_1/\partial t = -u_1 \partial u_1/\partial z - (1/\rho)\partial p_1/\partial z,$$
$$\partial u_2/\partial t = -u_2 \partial u_2/\partial z - \partial P/\partial z + (\Delta P/b)\partial R/\partial z, \quad \partial v_2/\partial t = -u_2 \partial v_2/\partial z - \Delta P/b, \tag{1}$$

together with the following equation for p_1 connecting the core and annular motions:

$$A_1 \partial^2 p_1/\partial z^2 + A_2 \partial p_1/\partial z + A_3 p_1 + A_4 = 0. \tag{2}$$

In the above representations, the coefficients A_1 to A_4 in (2) can be shown as functions of b, R, u_1, u_2, v_2, while P and ΔP are, respectively, like a mean pressure of p_1 and p_3 and a pressure difference between p_1 and p_3 when the surface tension is taken into account. In the above equations, the non-dimensional parameters based on the annular phase $\mathrm{Wb} = \rho_2 \bar{h}_2 \bar{u}_2^2/\sigma_2$, $\rho = \rho_1/\rho_2$ and $\sigma = \sigma_1/\sigma_2$ are introduced. Consequently, the problem can be reduced to solving the above simplified nonlinear equations.

[a)] Corresponding author. E-mail: yoshinag@me.es.osaka-u.ac.jp

NUMERICAL RESULTS

We consider a uniform semi-infinite jet ($0 \le z < \infty$) for $t = 0$ and the following velocity disturbances are fed at $z = 0$ for $t > 0$: $u_1 - \bar{u}_1 = u_2 - \bar{u}_2 = \eta \sin(\omega t + \phi)$. In the calculations, the breakup time t_b and distance z_b are examined for various ω in order to determine the critical ω_c which minimize t_b for each parameters ϕ, $\bar{u}_1/\bar{u}_2$ and Wb. Then, we can say that such ω_c are the most unstable in the sense of nonlinearity and different from those in the linear analysis. We first show in Fig.2 typical breakup profiles of the jet for different Wb when $\bar{u}_1/\bar{u}_2 = 1$, $\phi = 0$ and $\eta = 0.05$, where $\omega_c = 1.17$ in (a) for Wb $= 26.9$, while $\omega_c = 2.44$ in (b) for Wb $= 500$. From the Figure, it is found that the jet breaks up by pinching of the core phase for small Wb as in (a), while by disintegrating the annular phase for large Wb as in (b). Since the surface tension decreases as the increase of Wb, the breakup behavior may change from the radial pinching of the core phase to the disintegration of the annular phase due to the velocity shear instability. Therefore, it is found that the encapsulation of the core gas by the annular liquid fails when Wb becomes large.

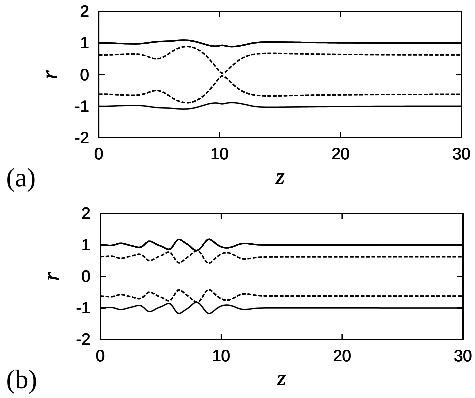

Figure 2. Typical breakup profiles for different Wb.

It is plausible that the disturbance with ω_c dominates the capsule formation due to the instability. In fact, this is confirmed for Wb $= 26.9$ from Fig.3 when the present numerical ω_c is compared with the previous results of the capsule formation period for different velocity ratios $\bar{u}_1/\bar{u}_2$. In this figure, the solid and broken lines denote, respectively, the numerical results for $\eta = 0.05$ when $\phi = 0$ and π together with the linear theory denoted by a dash-dotted line, while the open circles ◯ are the results of the experiment [2] and the open squares □ are of the phenomenological model [3, 4]. It is found that the experimental results (◯) are in better agreement with the present results (solid and broken lines) rather than the the phenomenological model (□). It is also found that these results are much larger than the linear prediction (dash-dotted line). Therefore, the present breakup and encapsulation are essentially nonlinear phenomena.

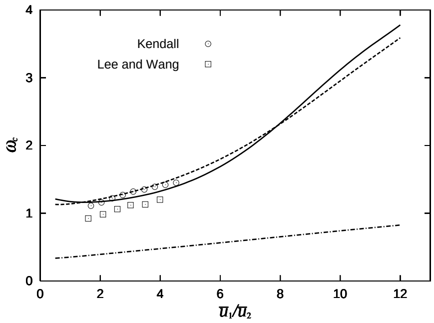

Figure 3. Capsule formation frequencies for different velocity ratios.

Furthermore, the sizes of produced capsules can be also evaluated from ω_c. Since the capsule producing period length λ is given by $2\pi\bar{u}_2/\omega_c$, the volume of the produced liquid capsule may consist of the equal volume of the annular jet whose wave length is λ. Figure 4 shows R_1, R_2 and $\lambda/2$ for different $\bar{u}_1/\bar{u}_2$. It is found that the experimental measurements of R_1 and R_2 denoted by ● and ◯ and the phenomenological model for R_2 by □ well agree with the present results. It is also found that the numerical R_1 and R_2 saturate and approach constant values for large velocity ratios. On the other hand, we note that a half of the formation period length $\lambda/2$ should be, at least, larger than the radius of the capsule, that is, $\lambda/2 > R_1$. Therefore, in order for well separated liquid capsule to be produced, we need a smaller velocity ratio than the critical one at which the curves of $\lambda/2$ intersect the curves of R_1 and R_2. From Fig.4, this critical value is estimated to be about 6, which is also confirmed from the fact that the experimental measurements are not obtained for larger values than this ratio.

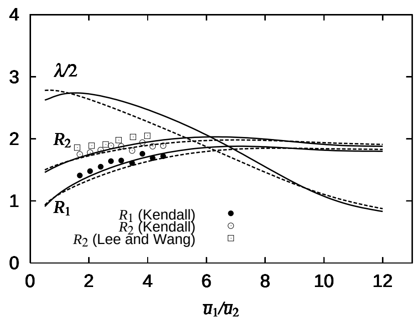

Figure 4. Inner and outer radii R_1, R_2 of the capsules and capsule formation period length $\lambda/2$ for different velocity ratios.

CONCLUDING REMARKS

We have shown that the most unstable ω_c can predict well the capsule formation periods and capsule sizes. This means that a particular disturbance with ω_c of any disturbances can survive during the breakup process and determines the breakup properties. Consequently, the mechanism of capsule formation in the compound liquid jet can be described by the nonlinear instability.

References

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