ROTATION OF ELLIPSOIDS IN CANONICAL FLOWS: EFFECTS OF PARTICLE AND FLUID INERTIA

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Summary Rotation of ellipsoidal particles is a fundamental physical problem when modelling multiphase flows with non-spherical particles. The dynamics of a particle are controlled by the Reynolds (Re) and Stokes (St) numbers, which measure the effects of fluid and particle inertia, respectively. Ellipsoids in two canonical flows: shear flow and extensional flow, are studied by numerical simulations. It has been shown earlier that as the particle inertia is increased, the rotation period in shear flow exhibits a transition from long to short that is fairly sharp in St. Here, these results are extended to the case of finite fluid inertia ($Re > 0$) . The transition is seen to move to higher St as Re is increased. For extensional flow, it is shown that the particle motion changes from direct alignment to damped oscillations as St is increased. The fastest alignment is found to occur for a non-zero value of the particle inertia. The insights from this (and similar) studies forms a knowledge base that is necessary to master in order to understand the motion of non-spherical particles in flows.

INTRODUCTION

The rotation of ellipsoids in canonical flows is a key ingredient for the understanding and modelling of multiphase flows with non-spherical particles. The width of applications as well as importance will be illustrated with three examples. The first example is ice crystals in clouds, their rotation controls collision rates and thus growth rates. The second example is plankton in the ocean; their rotation is an important factor for how they obtain their nutrition and thus for their growth. This growth, in turn, is an important factor for the carbon dioxide buffering of the oceans. The final example is cellulose fibres in a paper machine, and their rotation controls the fibre orientation, and thus the mechanical properties, of the final paper.

Here, the fundamental problem of the rotation of an ellipsoid in linear shear and extension flow will be studied. Assuming a body fixed coordinate system (x, y, z) and laboratory reference frame (x', y', z'), the ellipsoid is given by

$$x^2 + \left(\frac{y}{k_b}\right)^2 + \left(\frac{z}{k_c}\right)^2 = l^2 \tag{1}$$

so that the major axis of the ellipsoid has the length $2l$ and k_b, k_c are the aspect ratios (without loss of generality, it can be assumed that $k_b \leq 0$, $k_c \leq 0$) of the ellipsoid. The dynamics of such an ellipsoid in flow are governed by the two non-dimensional groups

$$Re = \frac{\dot{\varepsilon} l^2}{\nu}, \qquad St = \frac{\rho_p \dot{\varepsilon} l^2}{\rho_f \nu}, \tag{2}$$

where $\dot{\varepsilon}$ is the relevant flow-velocity gradient, ν the kinematic viscocity of the fluid, ρ_p and ρ_f the densities of the particle and fluid, respectively. The groups are called the Reynolds and Stokes number, respectively. They measure the effects of fluid (Re) and particle (St) inertia.

The flow around an ellipsoid for the case of $Re = 0$ (creeping flow) has been solved analytically[1], providing the torques on the particle in any linear flow, any orientation and for any particle rotation rate. These results were used to obtain the motion for light particles ($St = 0$), the so called Jeffery orbits. In spite of the fact that they have been known for almost 90 years, there are still new findings on particle dynamics being made based on these torques (e.g. [2,3,4]). For finite Re, one must rely to numerical simulations and this is also a very active research area[5,6].

In this work, both analytical[1] and numerical[6] methods are used to investigate the motion of ellipsoids in shear and extension. In particular, the effects of fluid inertia on the recently presented transition in rotation rate for high particle inertia[3] will be studied at finite Reynolds number. Furthermore, previous studies of particles in creeping ($Re = 0$) shear[3,4] will be extended to particle behaviour in extensional flow. This short paper focuses on the latter aspect.

METHODS

Two separate methods are used to model particle motion in this work. For the reference case of creeping flow, the analytical torques[1] are coupled to the equations of rotation for the ellipsoids. This gives a set of ordinary differential equations that can be studied theoretically and solved numerically. For the case of finite Re, the Lattice Boltzmann Method with discrete external boundary force[6] is used. In this short paper, the case of an ellipsoid in extenstional creeping flow is anayzed.

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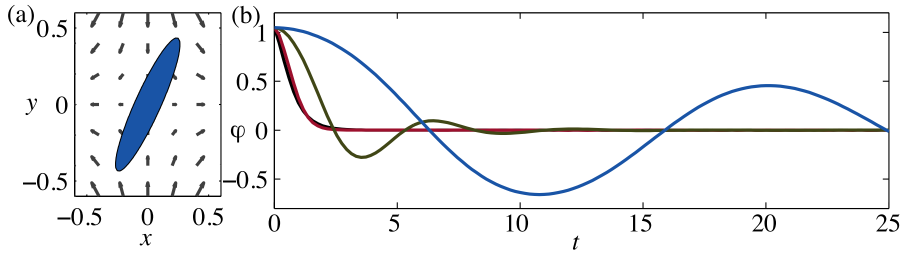

Figure 1. (a): Demonstration flow case: an ellipsoid ($k_b = k_c = 0.2$ in extensional flow. (b): Orientation φ as a function of time for $\varphi(0) = \pi/3$, the orientation shown in (a). $St/St_d = 0.1$ (black), 1 (red), 10 (green) and 100 (blue). Time is scaled with $\dot{\varepsilon}$.

For an inertial ellipsoid in extensional flow given as (u', v', w')=$(\dot{\varepsilon}x', \dot{\varepsilon}y', 0)$ with its longest axis in the $x'y'$-plane and φ denoting the angle between the x' and x axis (the flow situation is shown in figure 1 (a), the governing equation is $I_z\ddot{\varphi} = M_z$, where I_z and M_z are the moment of inertia and torque around the z-axis, respectively. Utilizing the analytical expression for the torque[1], the equation of motion is obtained as (with time scaled by $\dot{\varepsilon}$):

$$\frac{StF(k_b, k_c)}{20}\ddot{\varphi} + \dot{\varphi} + 2\frac{1 - k_b^2}{1 + k_b^2}\cos(\varphi)\sin(\varphi) = 0, \tag{3}$$

where $F(k_b, k_c) = k_c k_b(K_\alpha + k_b^2 K_\beta)$ and K_α are K_β are integral expressions including k_b and k_c[3,4]. Here, a few results from this equation will be shown and discussed.

RESULTS

First, small angles will be analyzed. It is observed that if equation (3) is linearized, an ODE of 2^{nd} order is obtained. The linearized equation (not shown) describes an oscillation with a natural frequency ω_n and damping factor ξ, where $\xi = 1$ gives critical damping:

$$\omega_n = \sqrt{\frac{40(1 - k_b^2}{StF(k_b, k_c)(1 + k_b^2)}} \qquad \xi = \sqrt{\frac{5(1 + k_b^2}{2StF(k_b, k_c)(1 - k_b^2}}. \tag{4}$$

Thus, heavy particles (high St) have a low frequency and low damping. In particular, there is a critical value of St for which $\xi = 1$ and the system is critically damped. We denote this value St_d and make the observation that at St_d, φ will approach 0 as fast as possible. For $St > St_d$, the particle will exhibit a dampened oscillation and for $St < St_d$, a particle released from rest will approach $\varphi = 0$ monotonically, albeit slower than for $St = St_d$.

Figure 1 (b) shows time traces of φ for an ellipsoid ($k_b = k_c = 0.2$) released from rest at $\varphi = \pi/3$ is shown. The findings above are verified. For $St = St_d$ (red), a fast approach to φ occurs, for lower St (black), the approach is finished somewhat later and for higher St (green and blue), the particle exhibits a damped oscillation with decreased damping and frequency as St is increased.

This short paper does not allow results for $Re > 0$ to be included. In short, fluid inertia is found to delay the transition in rotation rate[3] in shear to higher St.

CONCLUSIONS

The rich problem of ellipsoidal particles in shear and extensional flow is studied. The study starts off from the effects of particle inertia at $Re = 0$; modifications due to fluid inertia are included in the complete work. In particular, it has been shown that heavy particles can oscillate while aligning in extensional flow, and that the fastest alignment is obtained for a specific, non-zero particle inertia. The results provide fundamental knowledge regarding the behaviour of non-spherical particles in flows.

References

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