CURVATURE SINGULARITY AND FILM-SKATING DURING DROP IMPACT

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Summary We study the influence of the surrounding gas in the dynamics of drop impact on a smooth surface. We use an axisymmetric model for which both the gas and the liquid are incompressible; lubrication regime applies for the gas film dynamics and the liquid viscosity is neglected. A liquid sheet emerges from the zone of impact, skating above a thin gas layer. It is found that the thickness of the air film underneath this jet is smaller than the mean free path in the gas, such that the liquid film eventually wets the surface.

INTRODUCTION

Drop impact is present in many surface flows and diphasic dynamics. It is crucial to our understanding of atomization, ink-jet printing or deposition, as well as for environmental issues such as raindrop impact erosion or aerosol spreading. The general situation involves an almost spherical drop impacting on a dry or wet solid surface. A splash is observed for strong impact conditions, whereas a gentle spreading of the drop is seen otherwise. On a dry substrate as considered further on, the splash is characterized by a corolla that detaches from the solid substrate forming a corona shape from which droplets can eventually detach. This complex dynamics involves various parameters which can highly influence the transition between these two regimes: primarily viscous and capillary effects are invoked, quantified by the Reynolds and the Weber numbers respectively. The impacted surface is also important: in particular, the roughness of the surface can control the splash formation as enhanced with textured surfaces [5]. Recently, the surrounding gas, often neglected in these problems, has been shown to be crucial since the splashing observed at atmospheric pressure disappears when the ambient gas pressure is lowered [1]. Despite the few models already proposed to explain this striking effect, invoking in particular compressibility of the surrounding gas [1] or the entrapment of a (compressible) gas bubble by air cushioning [2, 3], a complete understanding for the mechanism of the splash formation is still lacking. The aim of this study is thus to disentangle the role of the surrounding gas for the drop impact on solid surfaces in the limits where the viscosity of the liquid of the drop can be neglected and where both fluid can be considered incompressible.

GOVERNING EQUATIONS AND NUMERICAL METHOD

We consider the impact of an incompressible non-viscous liquid drop of radius R, density ρ_l, surface tension coefficient γ, with vertical velocity V on a solid substrate (see figure 1). The gas is taken incompressible of density ρ_g, dynamic viscosity η; gravity is neglected and the geometry is supposed to remain axisymmetric at short time. The dominant effect of the gas lies in the dynamics of the viscous thin gas film underneath the drop. In this situation, the viscous boundary layer in the liquid created by the gas shear flow is small and the liquid velocity can be approximated by a potential flow while the dynamics of the gas layer follows the lubrication equation[2].

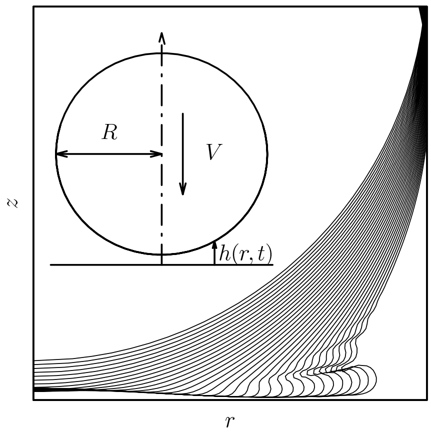

Figure 1. *Sketch of the impacting drop.*

The numerical method proceeds as follows : Laplace's equation for the velocity potential φ inside the drop is solved using a boundary integral method, the pressure is calculated through the lubrication equation in the gas layer and matched to a constant pressure outside this region, the interface and the velocity potential are advanced in time using the kinematic

[a)] Corresponding author. E-mail: duchemin@irphe.univ-mrs.fr condition and Bernoulli equation (in which the pressure is the lubrication layer underneath the drop) respectively. Surface tension is taken into account in Bernoulli equation through the term $\gamma\kappa$, where κ is the mean curvature of the interface. Once the lengths have been rescaled by R, velocities by V, densities by ρ_l and the gas pressure by $\rho_l V^2$, this set of equations introduces the two dimensionless numbers of the problem, the Weber and Stokes numbers : $\mathrm{We} = \rho_l R V^2/\gamma$ and $\mathrm{St} = \eta/\rho_l V R$.

RESULTS

In the absence of surface tension ($We \to \infty$), a finite-time singularity is observed [8]. It corresponds as $t \to t_0$ to a corner like interface located at $r_c(t) \to r_0$ where the curvature $\kappa_0(t)$ diverges as the film thickness $h_0(t)$ vanishes. To understand the properties of this singularity, we seek self-similar solutions of the form : $\tilde{h}(r,t) = h_0(t)H(R)$, $\tilde{p}(r,t) = p_0(t)P(R)$, and $\tilde{\varphi}(r,t) = \varphi_0(t)\Phi(R,Z)$, where $R = (r - r_0(t))/l(t)$ and $Z = z/l(t)$. Then, the balance between relevant terms in the equations allows us to highlight two different impact regimes :

Regime I : $h_0 \ll l$ leads to the observed numerical scalings : $p_0 \sim h_0^{-1/2}$ and $\kappa_0 \sim \mathrm{St}^{4/3} h_0^{-2}$. We thus deduce that this regime corresponds in fact to *thick* gas layer $h_0 \gg St^{4/3}$.

Regime II : $h_0 \gg l$ leads to $p_0 \sim \mathrm{St}^{-2/3}$ and $\kappa_0 \sim \mathrm{St}^{8/3} h_0^{-3}$. Consistently, this regime holds for *thin* gas layer $h_0 \ll \mathrm{St}^{4/3}$ and this analysis suggests that the self similar behavior observed in the numerics is valid only for large enough h_0. We predict thus another self-similar regime closer to the solid surface, not identified previously and never reached in all the numerical simulations so far [4, 7, 2, 6].

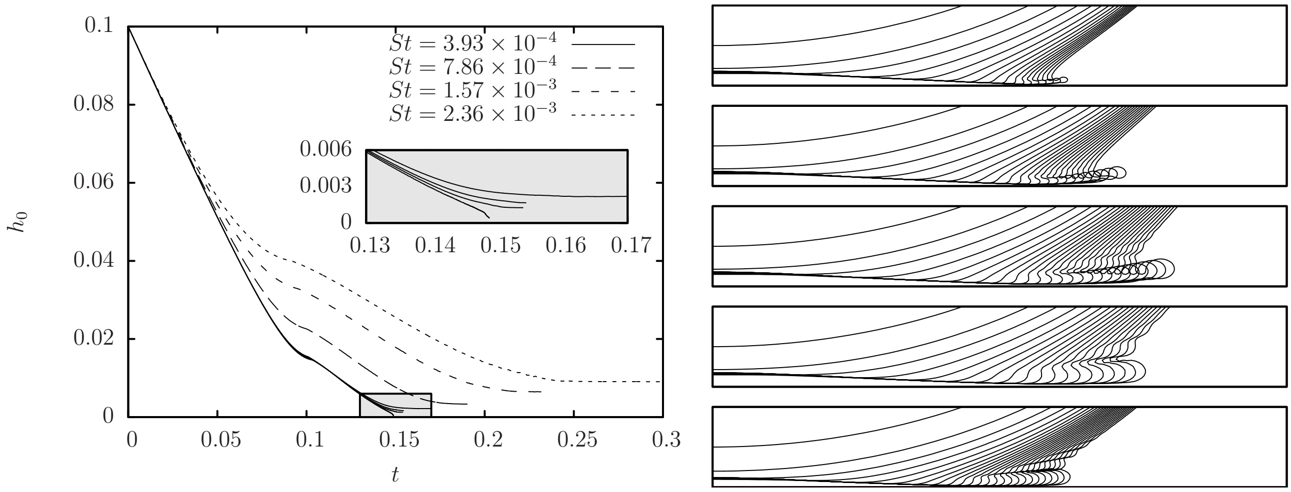

Figure 2. Left : *Minimum gas film thickness h_0 for four different values of the Stokes number ($St = 3.93 \times 10^{-4}, 7.86 \times 10^{-4}, 1.57 \times 10^{-3}, 2.36 \times 10^{-3}$), as a function of time. The zoom for $St = 3.93 \times 10^{-4}$ near the axis shows the evolution for different Weber number, $We = 47.5$, 95, 238 and ∞ from top to bottom.* **Right :** *Five different computations, with* $\mathrm{We} = 238$; 95; 47.5; 23.7 *respectively, from top to bottom and* $\mathrm{St} = 1.35 \times 10^{-3}$. *The bottom figure is for* $\mathrm{We} = 23.7$ *and* $\mathrm{St} = 6.29 \times 10^{-4}$.

In the realistic case (finite We), the minimal gas layer thickness (See figure 2) can be estimated thanks to the two different self-similar regimes exhibited for the singular case $\mathrm{We} \to \infty$. Indeed, balancing the capillary term $\mathrm{We}^{-1}\kappa$ with the singular pressure, in Bernoulli equation, gives for the gas layer, depending on the regime: $\mathrm{I} : h_0 \sim \mathrm{St}^{8/9}\mathrm{We}^{-2/3}$ and $\mathrm{II} : h_0 \sim \mathrm{St}^{10/9}\mathrm{We}^{-1/3}$ while the two behaviors cross for $\mathrm{We} \sim \mathrm{St}^{-2/3}$ (with high Weber number for regime II). Numerically, we find that the two exponents vary as follows $h_0 \sim \mathrm{St}^{0.9-1.}\mathrm{We}^{-0.33-0.4}$ which is in reasonable agreement with regime II. Computing the minimal gas thickness in the experiments [1] following the scaling laws for h_0 obtained above, we observe that it is of the order of a few $\mathring{A}$, much below the mean free path even at atmospheric pressure (~ 60 nm). Therefore, it is reasonable to consider that the liquid film eventually touches the solid so that the lubrication approximation made here is not valid anymore: instead we have a rapid expansion of a liquid sheet in contact with the solid.

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