EVOLUTION AND BREAKUP OF VISCOUS ROTATING DROPS

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Summary We study the evolution of a viscous fluid drop rotating about a fixed axis at constant angular velocity Ω or constant angular momentum L surrounded by another viscous fluid. The problem is considered in the limit of large Ekman number and small Reynolds number. The analysis is carried out by combining asymptotic analysis and full numerical simulation by means of the boundary element method. We pay special attention to the stability/instability of equilibrium shapes and the possible formation of singularities representing a change in the topology of the fluid domain. When the evolution is at constant Ω, depending on its value, drops can take the form of a flat film whose thickness goes to zero in finite time or an elongated filament that extends indefinitely. When evolution takes place at constant L, and axial symmetry is imposed, thin films surrounded by a toroidal rim can develop, but the film thickness does not vanish in finite time. When axial symmetry is not imposed and L is sufficiently large, drops break axial symmetry and, depending on the value of L, reach an equilibrium configuration with a 2-fold symmetry or break up into several drops with a 2 or 3-fold symmetry. The mechanism of breakup is also described.

INTRODUCTION

Studies of rotating drops date back to the original experiments performed by the belgian physicist J. Plateau [5]. These experiments, which have been recently conducted under zero gravity conditions during the ight of Spacelab 3 and at JPL [6], play an important role in many industrial processes such as polymer manufacturing and spinning drop tensiometry techniques [1], [4]. In this communication we will present a detailed description of the rotating drop problem, modelled by the Stokes system under low Reynolds number and low Ekman number assumptions. Dealing with the limit in which $Re \ll 1$ and $Ek \gg 1$, so that viscous forces dominate over inertial and Coriolis forces, we can approximate Navier-Stokes equations by the Stokes system for both the drop's fluid (i=1) and surrounding fluid (i=2):

$$-\nabla\Pi^{(i)} + \mu_i \Delta \mathbf{u}^{(i)} = 0 \quad \text{in } \mathcal{D}_i(t) \ , \tag{1}$$

$$\nabla \cdot \mathbf{u}^{(i)} = 0 \quad \text{in } \mathcal{D}_i(t) \ , \tag{2}$$

which is numerically solved in the next section using the boundary element method. Here, we use the reduced pressure

$$\Pi^{(i)} = p^{(i)} - \frac{1}{2}\Omega^2 r^2 \ , \tag{3}$$

where Ω is a dimensionless angular velocity. If the evolution takes place at constant angular momentum, then $L = I\Omega$ where I is the moment of inertial with respect to the axis of rotation.

DIFFERENT SHAPES

Depending on the values of angular velocity or angular momentum, the evolution may lead to equilibrium configurations with various possible shapes or to breakup in a particular way (see Figures). These include toroidal type I and type II (depending on whether a thin film lies "inside" the torus or not) shapes, peanut shapes (two balls connected by a bridge), expanding pizza shapes (films surrounded by a toroidal rim) and equilibrium ellipsoids (see also [2], [3]).

MAIN RESULTS

We have developed a numerical algorithm based on the boundary integral formulation of Stokes system. The numerical algorithm is adaptive and introduces automatically local refinement in the critical regions such as necks, where the drop is going to break up. Based on the numerical results and the analysis of the equations, we have described the evolution both at constant angular velocity Ω and constant angular momentum L. The different regimes found are summarized in tables I and II.
The values for Ω^* and L^* have been determined numerically, $\Omega^* = 4.3648...$, $L^* = 2.3755\ldots$. Other critical values for Ω and L (Ω_2, Ω_{ap}, Ω_{sp}, L_2, L_2^* and L_3^*) have been numerically computed for the particular case of a drop surrounded by a fluid with the same viscosity as the drop. We have verified that for other viscosity ratios up to 0.01, when the outer fluid viscosity is much smaller than the drop's viscosity, the qualitative picture does not change although the critical values can be slightly different.

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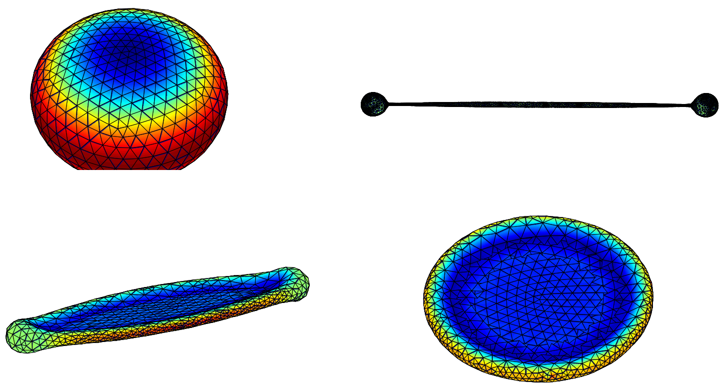

Figure 1. Shapes resulting from the evolution of the rotating drop at constant Ω: $\Omega < \Omega_2$ (top left), $\Omega_2 < \Omega < \Omega_{ap}$ (top right), $\Omega_{ap} < \Omega < \Omega_{sp}$ (bottom left) and $\Omega > \Omega_{sp}$ (bottom right). They are not at the same scale. The values of Ω_2, Ω_{ap} and Ω_{sp} are obtained numerically

Table 1. Axisymmetric evolution

Constant Ω	Constant L
$\Omega < \Omega^*$ Axisymmetric equilibrium	$L < L^*$ Axisymmetric equilibrium
$\Omega > \Omega^*$ Expanding *pizza* shape	$L > L^*$ Toroidal Type II

Table 2. 3D evolution

Constant Ω	Constant L
$\Omega < \Omega_2$ Axisymmetric equilibrium	$L < L_2$ Axisymmetric equilibrium
$\Omega_2 < \Omega < \Omega_{ap}$ Elongating filament	$L_2 < L < L_2^*$ stable *peanut* shape
$\Omega_{ap} < \Omega < \Omega_{sp}$ Asymmetric expanding *pizza* shape	$L_2^* < L < L_3^*$ Breakup (2-fold)
$\Omega > \Omega_{sp}$ Axisymmetric expanding *pizza* shape	$L > L_3^*$ Breakup (3-fold)

The approach, at constant $L > L^*$, to the toroidal type II solutions occurs at an $O(t^{-1/2})$ rate and the profiles for the interface present similarity features that we have described in detail. The elongating filaments at constant Ω reach an infinite length in finite time t_0 and the length blows up at a $O((t_0 - t)^{-1/2})$ and the interface profile approaches an explicit self-similar solution. Breakup at constant L is via an axisymmetric similarity profile. Finally, the axisymmetric expanding *pizza* shapes have a radius that grows at a $O((t_0 - t)^{-1/2})$, as shown by Howell et al. [4].

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