SIMILARITY SOLUTIONS FOR COALESCE OF TWO SESSILE DROPS

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Summary When coalescing, two viscous drops lying on a substrate are connected by a thin capillary meniscus bridge. At early times after the coalesce the size of the bridge is small compared to the drop radius, making the bridge dynamics independent of the characteristic drop size. Using this assumption, we simplify the lubrication equation describing the coalescence and identify a self-similar bridge dynamics. We find that the meniscus bridge viewed from the side grows linearly with time, while viewed from the top the bridge grow with a square root of time. The similarity profile compares very well with numerical simulations, as well as with side view experiments of coalescing drops.

INTRODUCTION

It is well known that for the drop break up that occurs in the dripping of a faucet, the radius of the neck scales linearly with time ($r_n \sim t$) for the viscous regime [1, 2], and in the inertial driven pinch-off it scales as $r_n \sim t^{2/3}$ [3]. These scaling laws can be derived by assuming that the closer to the pinch-off the shape of the neck becomes asymptotically self-similar. Similarly, one finds scaling laws for the coalesce of viscous or inviscid spherical drops [7, 8, 9]. By contrast, much less is known on the scaling of coalescing sessile drop, lying on a substrate. For viscous drops, Ristenpart *et al.* [6] found experimentally and numerically that the size of the bridge from the top view scales as $t^{1/2}$. However, there is still a lack of detailed understanding for this coalescence behavior. Here we present a similarity-solution based on the lubrication approximation for the meniscus bridge present in the coalescence of viscous drops.

SIMILARITY SOLUTIONS

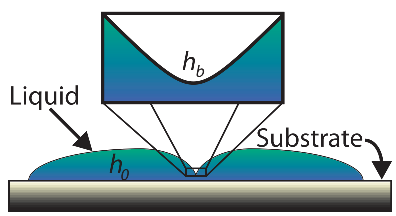

Figure 1. Sketch of the side view of the coalesce of two drops lying on a substrate. Both drops are the same size and the meniscus bridge connecting them has a height h_b

According to Ristenpart *et al.* [6], during the coalescence of the drops lying on a substrate, shown in Figure 1, the main direction of the flow is parallel with a line drawn between the centers of the two drops. Using this information, it is possible to describe the problem using the one-dimensional lubrication approximation equation.

$$\frac{\partial h}{\partial t} + \frac{\gamma}{3\eta}\frac{\partial}{\partial x}\left[h^3\left(\frac{\partial^3 h}{\partial x^3} - \frac{\rho g}{\gamma}\frac{\partial h}{\partial x}\right)\right] = 0, \tag{1}$$

in which h is the height of the profile, η is the viscosity, γ is the surface tension, ρ is the fluid density and g is the gravity acceleration. In order to scale the equation we used the capillary length $l_c = (\gamma/\rho g)^{1/2}$, the characteristic time $t_c = 3\eta\ l_c/\theta_{eq}^3\gamma$, and the dimensionless horizontal and vertical directions $x = l_c\ X$, $h = l_c\ \theta_{eq}\ H$ and time $t = t_c\ T$. Here, θ_{eq} is the equilibrium contact angle. It is also possible to assume that close to the meniscus bridge the gravitational effects are negligible $\left(\frac{\partial^3 H}{\partial X^3} \gg \frac{\partial H}{\partial X}\right)$, since for early times after the coalescence the bridge is a singular point, in which the shape of the meniscus is independent of the boundary conditions, *i.e.*, the size of the drops, or the gravity. This assumption leads to the simplified equation

$$\frac{\partial H}{\partial T} + \frac{\partial}{\partial X}\left(H^3\frac{\partial^3 H}{\partial X^3}\right) = 0. \tag{2}$$

In order to find the universal shape of the profile, we proposed the similarity solution with the form

$$H\left(X,T\right) = a\ T\ \mathcal{H}\left(\xi\right), \quad \text{with} \ \ \xi = \frac{X}{a\ T}, \tag{3}$$

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in which a is a dimensionless propagating velocity in vertical and horizontal direction. By substituting equation 3 in 2 we get the ordinary differential equation

$$\mathcal{H}(\xi) - \xi\mathcal{H}'(\xi) + \frac{1}{a}\frac{d}{d\xi}\left[\mathcal{H}^3(\xi)\mathcal{H}'''(\xi)\right] = 0. \tag{4}$$

Using the appropriate boundary conditions, we solved numerically this equation and found the universal shape of the meniscus bridge.

COMPARISON WITH NUMERIC SIMULATIONS AND EXPERIMENTS

In order to test the validity of the profile we performed finite element method simulations and side view experiments of the coalescence of spreading drops. For the numeric simulations the initial condition is the steady solution of two independent drops linked by a smooth interface. The results of the numerics was compared with the theory are shown in Figure 2. On the left, we show the comparison between the scaled numerical profiles for different moments in time, and the profile from the self-similarity solution. On the right we show the height of the bridge (h_b) from the similarity and the numerical solution. In both plots it can be seen that the numerical profiles overlap with the similarity profile for the earliest times. Preliminary experiments reveal that the side-view profiles indeed follow the same similarity solution.

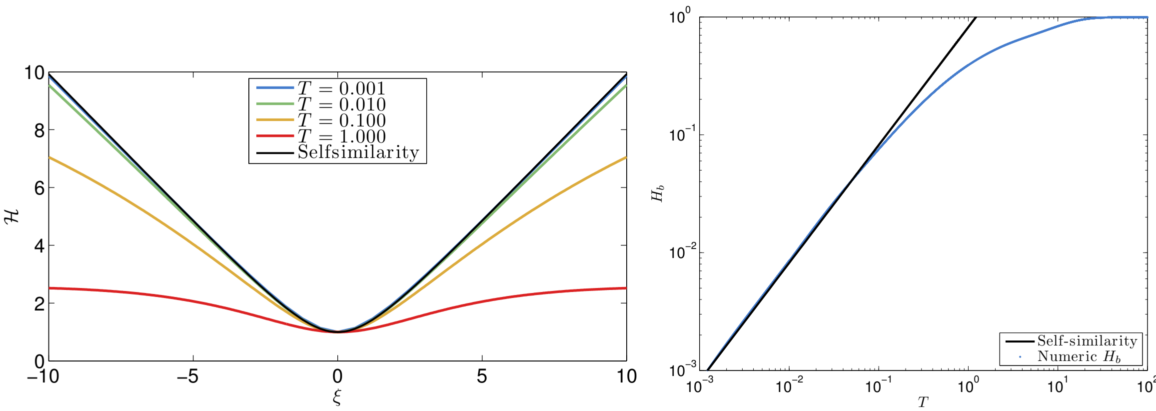

Figure 2. Left: Comparison between the similarity profile and the numeric profile for different moments in time. Right: Comparison of the height of the bridge (H_b) as a function of time between the numeric simulation (blue), and the self-similarity height.

CONCLUSIONS

Using the lubrication approximation we demonstrated that there is a universal shape for the meniscus bridge that links two droplets lying on a substrate. By proposing a similarity scaling we found an ordinary differential equation that represents the meniscus bridge during the early times after the coalesce. This equation was solved numerically, and the resulting profile compares very well with numerical solutions. Preliminary experiments appear to confirm this scenario for coalescence of very viscous drops on a surface.

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