PHASE CHANGE IN POROUS MEDIA UNDER CONSTANT HEAT FLUX

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Summary This paper introduces a one-dimensional analytical solution capable of predicting the temporal evolution of solid-liquid phase interface in two-phase (continuous and discrete phases) heterogeneous materials with constant heat flux thermal boundary condition. Numerical simulations based on the finite difference method are performed for validation. Analytical and numerical solutions show that the presence of gaseous pores significantly enhances the solidification process due to the decreased effective density of the bulk material.

INTRODUCTION

Phase change problems associated with solidification or melting involve a wide variety of practical applications under different thermal boundary conditions [1]. Numerous efforts have been devoted to investigating the temporal evolution of phase interface, temperature distribution in the solidified or molten layer, full solidification or molten time, and other issues in phase change problems [2]. Due to mathematical complexities in problems with moving boundaries (interfaces), few exact analytical solutions for phase change problems are presently available including Neumann's solution for the solidification of a semi-infinite slab with the first kind thermal boundary (constant temperature) and El-Genk's solution for the freezing of a semi-infinite liquid with the second kind thermal boundary (constant heat flux) [3]. Whilst these exact solutions dealing with one-dimensional semi-infinite phase change are only applicable to homogeneous materials, most materials exploited in engineering applications contain more than two phases. This study aims to demonstrate that the temporal evolution of the interface between different phases under constant heat flux conditions can be predicted analytically. The analysis is limited to two-phase heterogeneous materials consisting of dense materials and randomly spaced gaseous pores, with close-celled aluminum foams (Fig. 1(a)) fabricated via the foaming route chosen as the prototype. To obtain aluminum foams having high porosity, it needs not only to precisely control their porosity by using the relationship between porosity and foaming time, but also to solidify the melt in time during the plateau stage on the porosity-time curve to control the pore size and uniformity of pore distribution [4].

ANALYTICAL AND NUMERICAL APPROACHES

Consider the one-dimensional solidification problem as illustrated schematically in Fig. 1(b). Idealized circular pores having zero thermal conductivity and density are randomly distributed in the continuous phase (water), with pore sizes varying in the range of 0.635 to 5 mm. The bulk porous medium has the total length of H and an infinite thickness (for simplicity). Initially, the continuous medium containing the non-conducting pores is at its melting temperature T_m. At time $t \geq 0$, a constant heat flux (q_w'' = constant) is applied at surface $x = 0$ (Fig. 1). All the relevant physical properties of the material (in either liquid or solid state) are taken as invariant in temperature, time, and space. It is further assumed that the transfer of heat within the solidified layer is solely by conduction, with radiation and natural convection ignored.

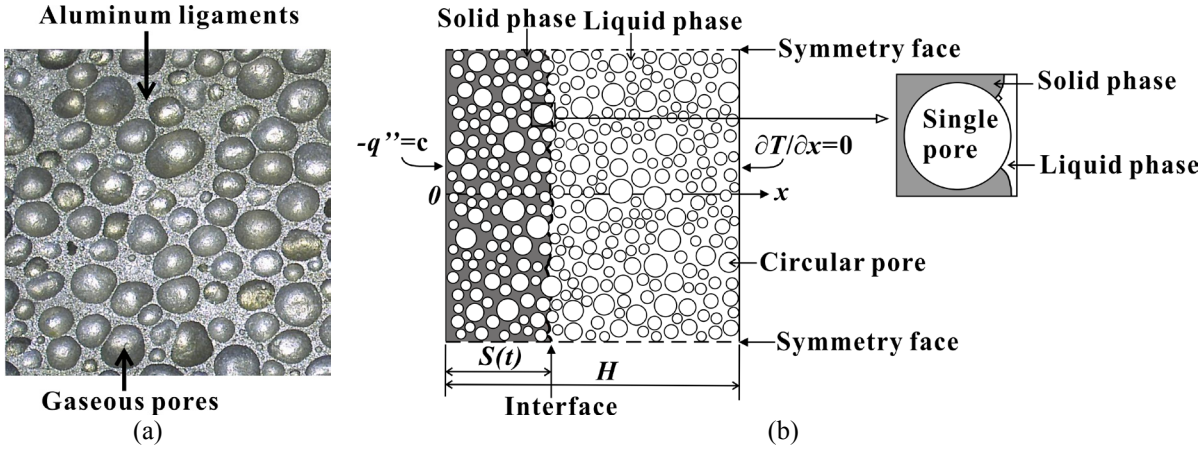

FIG. 1 Solidification model for closed-cell aluminum foam: (a) close-celled aluminum foam with circular pores; (b) one-dimensional computational model at $Ste'' = 1.0$ and $\tau = 0.305$.

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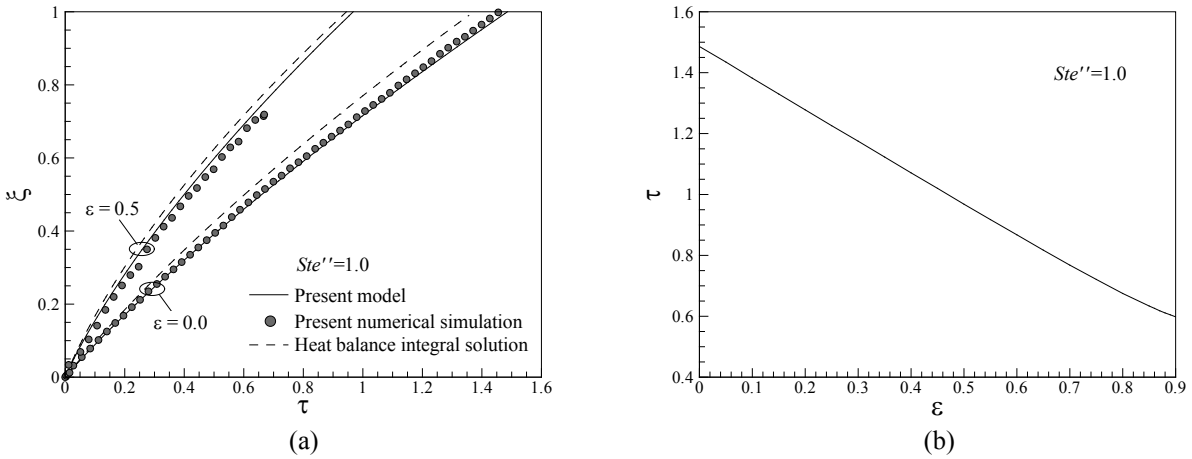
Given the random distribution of the discrete phases in the continuous medium, the phase interface location may be obtained by combining El-Genk's model [3] and Bauer's effective thermal conductivity model [5], as:

$$\frac{\mathrm{d}\xi}{\mathrm{d}\tau} = \frac{1}{1-\varepsilon} Ste'' \mathrm{erfc}\left(\frac{\xi}{2\sqrt{(1-\varepsilon)^{3\beta/2-1}\tau}}\right) \quad (1)$$

where ξ is the dimensionless interface location defined as $\xi = S(t)/H$, τ represents the dimensionless time defined as $\tau = \alpha t/H^2$, ε is the porosity, β is the pore shape factor, *Ste''* is the Stefan number for constant heat flux thermal boundary defined as $Ste'' = c_p H q''/(kL)$, and erfc(x) is the complementary error function. A snapshot of the solid-liquid interface shown in Fig. 1(b) demonstrates that the solid-liquid interface is not perpendicular to the x-axis, due to the heterogeneity of the porous medium containing randomly distributed pores. Further, locally, the interface is normal to the boundary of the circular pores due to the assumption that the pores are non-conducting.

DISCUSSION OF RESULTS

The temporal evolution of phase interface for dense ($\varepsilon = 0.0$) and porous ($\varepsilon = 0.5$) materials obtained by the present analytical model is shown in Fig. 2(a). To validate the model predictions, data from both numerical simulations and heat balance integral solutions are also included in Fig. 2(a), with good agreement observed. The solidification layer is monotonically but non-linearly thickened with time. At a given time, the solidified layer for the porous medium ($\varepsilon = 0.5$) is thicker than that for the dense material. This indicates that the presence of non-conducting circular pores decreases the effective density ($\rho_e = (1-\varepsilon)\rho_c$) of the bulk material, resulting in a faster phase change than that occurring in the dense material ($\varepsilon = 0.0$). It has been established that the effective density is sharply decreased due to the inclusion of the pores whilst the latent heat and cooling heat flux remain constant. Figure 2(b) displays the predicted full solidification time as a function of porosity for *Ste*" = 1.0. The time for full solidification is seen to decrease monotonically but non-linearly as the porosity is increased. This implies that the porosity plays a crucial role in the solidification of porous media: even a low porosity can significantly accelerate the solidification process.



(a) (b)

FIG. 2 Solidification of porous medium under fixed cooling condition (*Ste''*=1.0): (a) temporal evolution of solidification front; (b) full solidification time plotted as a function of porosity.

CONCLUSIONS

A new analytical model capable of predicting the one-dimensional temporal evolution of solid-liquid interface in two-phase (continuous and discrete phases) heterogeneous materials has been introduced under constant heat flux thermal boundary conditions. It is demonstrated that the inclusion of non-conducting pores significantly accelerates the solidification process due to the decreased effective density of the bulk material.

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