# SLENDER BODY THEORY FOR THE GENERATION OF MICROEMULSIONS AND MICROBUBBLES WITH MEDICAL APPLICATIONS.

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Summary We report experiments in which a flow rate $Q_i$ of a fluid with a viscosity $\mu_i$ discharges into an immiscible liquid of viscosity $\mu_o$ that flows in parallel with the axis of the injector. When the outer capillary number verifies the condition $Ca_o = \mu_o U_o/\sigma \geqslant 5$, where $U_o$ and $\sigma$ indicate, respectively, the outer velocity and the interfacial tension coefficient and if the inner to outer velocity ratio is such that $q = U_i/U_o = Q_i/(\pi U_o R_i^2) \ll 1$, with $R_i$ the inner radius of the injector, a jet is formed with the same type of cone-jet geometry predicted by the numerical results in [1]. Moreover, thanks to the information extracted from numerical simulations of boundary-integral type and using the slender-body approximation in [2], we deduce a third-order, ordinary differential equation that predicts the shape of the jet and the sizes of the drops generated.

## INTRODUCTION

The several excellent reviews existing on the subject of drop formation (see [3, 4] and references therein) reveal that the generation of microemulsions possesses many different applications in medicine, pharmacy, material science and food industries. One of the many different procedures to generate drops with sizes the micron range is based on a simple device, suggested by the numerical results of [1], in which two immiscible fluids coflow coaxially in the creeping flow limit. These authors realized that, under appropriate conditions, a tiny jet of liquid is ejected from the tip of a conical-shaped drop attached to the injector and named this new regime tip streaming. The geometry of the cone to jet transition, which is quite similar to that observed in the electrospray of charged liquids occurs, surprisingly, in the absence of surfactants. In this paper we firstly perform an exhaustive experimental study of the coflowing configuration operated in the tip streaming regime (see figure 1). From the point of view of applications, the simple coflow configuration operated in

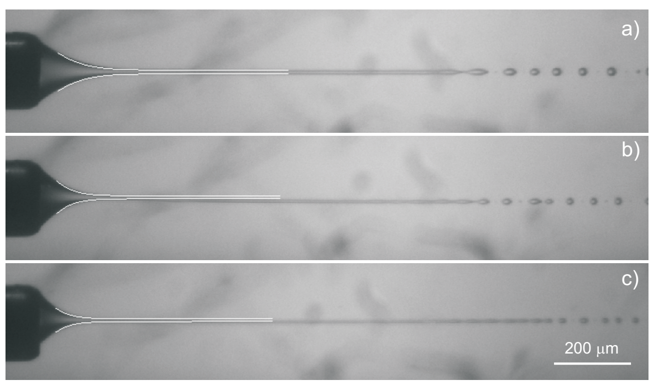


**Figure 1.** Experimental images obtained using silicon oil of $10\,cp$ as the inner fluid and glycerine as the outer ($\lambda = 10^{-2}$). The continuous white line indicates the theoretical jet shape obtained integrating the third order equation for the jet shape deduced using the extension deduced here to the slender body theory due to Taylor [2]. The values of the control parameters in each of the three cases shown are: (a) $Ca_o = 5$, $q = 10^{-2}$; (b) $Ca_o = 9$, $q = 6 \times 10^{-3}$; (c) $Ca_o = 12$, $q = 5 \times 10^{-3}$.

the tip streaming regime of figure 1 is an efficient method for the production of concentrated micro-emulsions composed of drops with a narrow size distribution, as it is shown in figure 2. Based on the information gathered from experimental observations and numerical simulations, we have developed a theory that accurately predicts the shapes of the jets and the sizes of the drops formed.

## EXPERIMENTS AND COMPARISON WITH NUMERICAL SIMULATIONS AND THEORY

The formation of a tiny jet ejected from the tip of a drop attached at the exit of the injector has been experimentally observed in the limit of low Reynolds numbers and for low values of the Bond number. Thus, this physical situation is controlled by only three dimensionless parameters, namely, the velocity ratio $q = Q_i/(\pi R_i^2 U_o)$, the viscosity ratio $\lambda = \mu_i/\mu_o$ and the capillary number $Ca_o = \mu_o U_o/\sigma$, where the subscripts $i$ and $o$ refer to the inner and outer streams respectively. Experiments have been performed using, as working fluids, different silicon oils with viscosities of $\mu$ =10, 100 and 1000 cp, glycerine, distilled water and air. Consequently, the inner to outer viscosity ratio $\lambda = \mu_i/\mu_o$ can be

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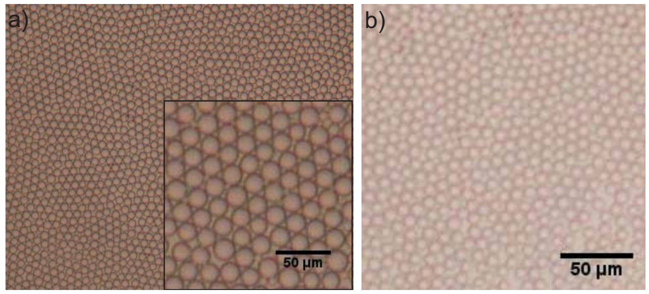


**Figure 2.** Concentrated emulsions using silicon oil with a viscosity of $10\,cp$ as the inner fluid and a $16\,mM$ SDS (Sodium Dodecyl Sulfate) glycerine and water mixture with a viscosity of $100\,cp$ as the outer one. Here, $\sigma = 55\,mN/m$. (a) Drop diameter, $D_d = 19\,\mu m$, $Ca_o = 5 \times 10^{-1}$, $q = 7 \times 10^{-2}$. The inset shows the crystal-like structure of the resulting emulsion; (b) $D_d = 8\,\mu m$, $Ca_o = 5 \times 10^{-1}$, $q = 8 \times 10^{-3}$. Concentrated emulsions composed of drops with sizes close to 1 $\mu m$ can be also generated. Nevertheless, due to the smallness of these drops, the images corresponding to these types of emulsions appear blurred and thus, are not shown here.

varied between $10^{-1}$ and $10^{-5}$. We have performed numerous experiments in which we have systematically varied each of the three parameters keeping the other two constants and the results can be summarized as follows:

- If $q$ decreases, the jet diameter decreases and so does the drop size.
- If $Ca_o$ increases above the threshold value that characterizes the dripping-jetting transition, the jet breakup length is approximately proportional to $Ca_o$ and the cone to jet geometrical transition becomes less slender. Drop size is not affected by changes in $Ca_o$.
- If $\lambda$ decreases, the cone to jet geometrical transition becomes more slender. For extremely low values of $\lambda$, the jet is unsteady and the size of the bubbles generated is not uniform.

In order to predict the shape of the liquid jets and the sizes of the drops produced, we compare the experimental results with those calculated numerically by means of a Boundary Element code for the Stokes equations. The analysis of the numerical results, which reproduce the experiments, suggests that the geometry of the cone to jet transition can be described by means of a theoretical approach based on the classical slender-body theory due to Taylor [2]. A third-order differential equation for the jet shape is deduced, integrated and compared with experiments, finding that our theoretical approach is able to predict the experimental observations for the whole range of experimental conditions in which tip streaming occurs (see figure 1). Most interestingly, the influence of the geometry of the injector system on the jet shape and drop size enters explicitly into the third-order differential equation; this extension to the classical slender-body theory due to Taylor [2] is our most relevant contribution. Finally, the size of the drops generated are very well predicted by the classical stability analysis due to Tomotika [5].

## CONCLUSIONS

In this presentation we have performed an exhaustive experimental, numerical and theoretical study on the so called tip streaming regime after [1]. The influence of the geometry of the injector system on the jet shape and drop size enters explicitly into the third-order differential equation deduced using the slender-body approximation through two functions that can be easily calculated numerically. Therefore, our theory can be used as an efficient tool for the design of new emulsification devices.

## References

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