A MOVING MESH METHOD FOR THREE-PHASE FLOWS WITH TRIPLE JUNCTION POINTS

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Summary We present a moving mesh method for three-phase flow problems with triple junction points. This method employs a body-fitted unstructured mesh where the interfaces between phases are lines of the mesh, and the triple junction points are nodes of the mesh. Dynamic boundary conditions on the interfaces and the triple points are incorporated accurately in a Finite-Element formulation based on Taylor-Hood element. The method is used to investigate the rising of a liquid drop with an attached bubble in a lighter liquid.

INTRODUCTION

Three phase flow problem has generated wide interest among theoretical researchers [1]. It is also of importance in industrial applications. This work is devoted to a novel ALE (Arbitrary Lagrangian Eulerian) method for three-phase flows with emphasis on the numerical treatment of triple junction points. In this method, the interfaces between different phases are lines of the mesh system, and the triple junction points (if they exist) are mesh nodes. The explicit representation of interfaces allows accurate approximations of boundary conditions on the interfaces and the triple points. Fig. 1 shows one such triangulation of a floating lens, where the three thick solid lines represent mesh lines on the interfaces and the thin dashed-lines interior mesh lines of the triangulation. A triple junction point where the three interfaces meet is located on the node represented by a black square. The three interfaces divide the computation domain into different regions each of which contains only one phase. Maintaining a valid and high quality mesh is crucial for the success of a moving mesh method; we achieve this goal by re-arranging locally the computational mesh system at each time-step [2].

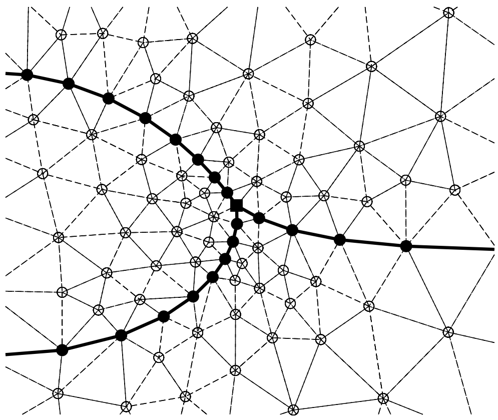

Figure 1. Triangulation of the computational domain of a floating lens: the empty circles ◦ present interior mesh nodes, the black circles • interface nodes and the black square the triple point. The thick solid lines are interfaces and the dashed-lines interior mesh lines of the triangulation.

GOVERNING EQUATIONS AND NUMERICAL METHOD

We consider flows of three different liquid phases L_i with density ρ_i and viscosity μ_i, where $i = 1, 2, 3$. The surface tension coefficient on an interface between two different liquids L_i and L_j is noted as $\sigma_{(ij)}$. The three-fluid flows are in general highly non-linear. They are governed by the incompressible Navier-Stokes equations; the continuity equation reads:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

and the momentum equation

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot (\mu \mathbf{D}) + \rho \mathbf{g}, \tag{2}$$

where $\mathbf{u}$ is the velocity, p is the pressure, $\mathbf{D}$ the strain rate tensor and $\mathbf{g}$ the gravity acceleration. The boundary condition on the interfaces expresses the force balance between the surface tension and the stress:

$$[(-p\mathbf{I} + \mu \mathbf{D}) \cdot \mathbf{n}]_-^+ = \sigma \kappa \mathbf{n}, \tag{3}$$

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where κ is the curvature, and $\mathbf{n}$ the normal vector of the interfaces. The triple junction points are subject to the pulling forces from three interfaces. Since they are without mass, the vector resultant of the three forces must be zero, which is, in mathematical term,

$$\sigma_{12}\mathbf{t}_{12} + \sigma_{23}\mathbf{t}_{23} + \sigma_{31}\mathbf{t}_{31} = 0, \tag{4}$$

where $\mathbf{t}_{ij}$ is the unit tangent vector at the triple point on the interface between L_i and L_j. Our numerical method is an extension of the ALE method for free surface flows in [2]. We discretize the above governing equations with a finite-element method using *P2-P1 Taylor-Hood* element. In each element, the velocity is approximated by a quadratic function, while the pressure by a linear function. In our method, the *local cubic spline* algorithm is used to obtain a continuous smooth parametrization of the interfaces. The zero resultant condition on the triple points is reinforced at each time step using a *Newton-Raphson* method. Finally, we solve the resulting algebraic system by an augmented Lagrangian technique with a *Uzawa* iterative algorithm.

RISING OF GAS-LIQUID DROPLET

The upward migration of a denser oil liquid drop attached to a gas bubble inside a capillary filled with water is of importance in industrial applications. This work is motivated by the extraction of the dense oil in porous media and can be used to determine efficient parameters of the physical process. We have studied the rising of a gas-liquid combined body using fully numerical simulations. Fig. 2 shows such a flow simulation. In this figure, the black area represents the heavier liquid and the white area under the wide dashed lines the gas. The physical parameters are $\rho_1 = 1.0$ (the lighter liquid), $\rho_2 = 0.001$ (the gas) and $\rho_3 = 1.1$ (the heavier liquid), $\mu_1 = \mu_3 = 1.0$, $\mu_2 = 0.01$. $\sigma_{(1,2)} = 0.5$, $\sigma_{(2,3)} = 1.0$, $\sigma_{(3,1)} = 0.6$ and $g = -1.0$. We are interested in the flow situation where the the gas-liquid droplet is usually small, and the Reynolds number $Re = 0$. At the initial instance, the combined gas-liquid body is a circular droplet of radius 1.0. The bubble occupies about 15.6% of the body volume. The interface between the gas and liquid is flat. As the triple point is not at equilibrium, the fluid system adjusts itself abruptly at the first time step. Apart from deforming under the surface tension to reach its natural shape with zero gravity, the combined body is also elongated as it moves upwards under the gravitational force. It reaches a steady rising speed of about 0.20 at time $t = 150$. Computations on several meshes with different size were performed to ensure that the result obtained is independent of the meshes used. One of useful applications of this numerical investigation can be the parametric study on the rising speed as function of volume percentage of the gas within the droplet. Since the density of the dense liquid is not much different from that of the background liquid, but three order of magnitude of that of the gas, a small amount of gas attached to the liquid droplet will be able to drag it upwards. The size of gas bubble may be tiny and therefore difficult to measure accurately. By measuring the steady rising speed and the liquid droplet volume, the gas volume can be obtained from the suggested parametric study.

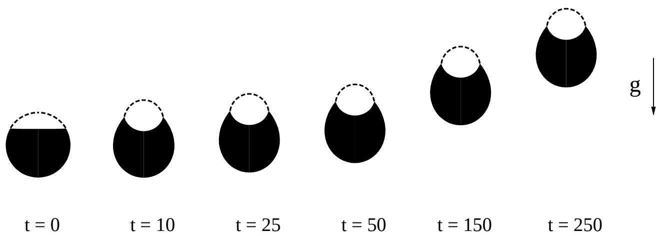

Figure 2. A sequence of vertical migration of gas-liquid combined body. The black area represents the liquid and the white area under the wide dashed lines the gas. The steady rising speed is about 0.20.

CONCLUSIONS

We have developed an ALE (Arbitrary Lagrangian Eulerian) moving mesh method suitable for solving two-dimensional and axisymmetric three-liquid flow problems with triple junction points. The method is used successfully to investigate the rising of a liquid drop with a attached bubble in a lighter background liquid.

References

[1] Pierre-Gilles de Gennes, Francoise Brochard-Wyart, and David Quere. *Capillarity and Wetting Phenomena: Drops, Bubbles, Pearls, Waves*. Springer, 2003.

[2] J. Etienne, E. J. Hinch, and J. Li. A lagrangian-eulerian approach for the numerical simulation of free-surface flow of a viscoelastic material. *J. Non-Newtonian Fluid Mech.*, *136*:157–166, 2006.