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DYNAMICS AND RHEOLOGY OF ELASTIC PARTICLES IN AN EXTENSIONAL FLOW

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Summary We investigate the dynamics of elastic particles in a viscous extensional flow under Stokes flow conditions. These particles are assumed to be homogeneous, incompressible, neo-Hookean solids with an initially stress-free ellipsoidal shape. When subjected to an extensional flow, an ellipsoidal particle will be stretching and rotating simultaneously. Eventually it deforms into another stable, ellipsoidal shape with the initial major axis aligned with the extension direction. In particular, the steady-state solutions may not exist when the particle stiffness is lower than a certain critical number. By using the solution of a single particle, the macroscopic rheological properties are evaluated for a dilute suspension of such elastic particles in an extensional flow. Counterintuitively, soft particles tend to increase the extensional viscosity of the suspension.

INTRODUCTION

The motion of soft solid particles in viscous fluid flows is an important topic in fluid mechanics. Among various types of soft particles, many of them can be effectively treated as homogeneous elastic solids [1, 2]. When subjected to viscous flows, the deformation of these "homogeneous" elastic particles is determined by the balance of the hydrodynamic force on the particle surface and the elastic resistance force within the solid. Often special numerical techniques [3] are required to solve the nonlinear fluid-particle interactions. We have developed a large deformation theory to characterize the finite-strain, time-dependent response of homogeneous, incompressible, neo-Hookean particles in a viscous fluid under Stokes flow conditions [4]. We have shown that as long as the far-field velocity gradient of the flow is constant, an elastic ellipsoidal particle will undergo a homogeneous deformation with a uniformly distributed physical field (stress, strain-rate, pressure, etc.), which is analogous to the classical Eshelby problem in elasticity [5]. The theory has been used to describe the rheology of a dilute suspension of initially spherical [4] and ellipsoidal [6] neo-Hookean particles in an unbounded simple shear flow. Here, we consider the dynamics of such particles in an extensional flow.
We start with the governing equations derived from the large deformation theory in [4] and focus our presentation in the case of two dimensions with initially unstressed elliptic particles. The complete particle dynamics can be described by the following system of ODEs [4],

$$\left.\begin{aligned} &\dot{\xi}^p_{11} = \xi^p_{12}\left[2(\omega+1)^2\sin 2\theta + (\omega^2+1)\xi^p_{12}\right]\Big/(\omega^2-1), \quad \dot{\xi}^p_{12} = \left(\xi^p_{11} - (\Lambda+1/\Lambda)/2G\right)\left[2(\omega+1)^2\sin 2\theta + (\omega^2+1)\xi^p_{12}\right]\Big/(\omega^2-1), \\ &\dot{\omega} = -2\omega(\omega+1)^2\cos 2\theta\Big/(\omega^2+1) + \omega\xi^p_{11} - \omega^3(\Lambda+1/\Lambda)\Big/G(\omega^2+1), \quad \dot{\theta} = \left[(\omega+1)^2\sin 2\theta + \omega\xi^p_{12}\right]\Big/(\omega^2-1), \end{aligned}\right\} \tag{1}$$

where ξ^p_{11} and ξ^p_{12} are components ("11" and "12") of a modified stress tensor ξ^p; ω is the particle aspect ratio; θ is the particle orientation. The over dot represents the time derivative of the variables. The equations are nondimensionalized with the characteristic length scale d_p (mean particle diameter), the time scale $\dot{\gamma}^{-1}$, the velocity scale $\dot{\gamma}d_p$, and the pressure and stress scale $\mu\dot{\gamma}$ (where μ is fluid viscosity and $\dot{\gamma}$ is the far-field strain rate). The elastic deformation is characterized by the shear modulus η, so that the dimensionless parameter $G = \mu\dot{\gamma}/\eta$ is a measure of the viscous force in the fluid relative to the elastic force in the solid. The dynamics of the particle also depends on its initial aspect ratio Λ.

RESULTS AND CONCLUSIONS

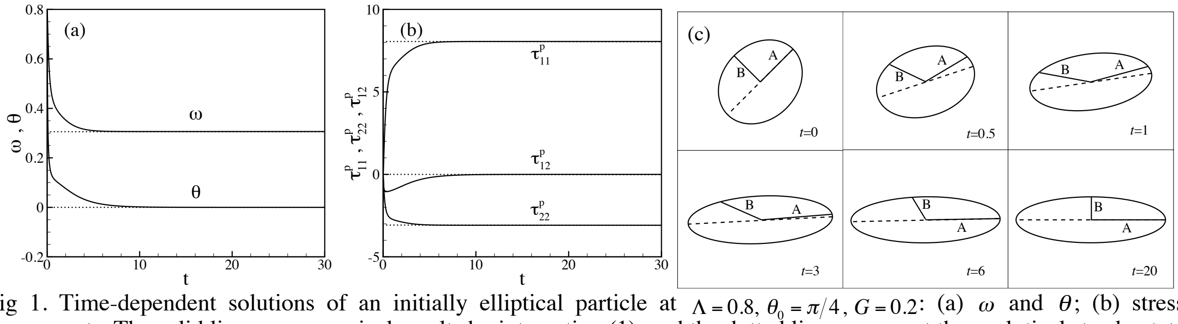

Fig 1. Time-dependent solutions of an initially elliptical particle at $\Lambda = 0.8$, $\theta_0 = \pi/4$, $G = 0.2$: (a) ω and θ; (b) stress components. The solid lines are numerical results by integrating (1), and the dotted lines represent the analytical steady-state solutions of (1). (c) Snapshots of the contour and material-lines at six time instants.

The system (1) can be numerically integrated with certain initial aspect ratio Λ and orientation θ_0 for a given value of G. For $\Lambda = 0.8$, $\theta_0 = \pi/4$, $G = 0.2$, the time-dependent solutions are shown in Fig. 1. The evolutions of ω and θ in 1(a) show that when subjected to the extensional flow, the particle becomes elongated as ω decreases to a steady value, and the major axis of the particle rotates to align with the extension direction at $\theta = 0$. In Fig. 1(b), the first normal stress component τ^p_{11} dominates due to the hydrodynamic stretching along the extension direction, and the shear stress component τ^p_{12} is identically zero at steady state. Considering the fact that an initially straight material line remains straight in a linear deformation field, we can visualize the particle motion by tracking two special material lines that are

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initially aligned with the semi-major (line *A*) and semi-minor (line *B*) axes, as shown in Fig. 1(c) where the instantaneous major axis of the particle is marked by a dashed line. As shown in Fig.1(c), both lines *A* and *B* are stretched and rotate, and eventually align with the major and minor axes, respectively, at the steady state.

For a given particle with an initial aspect ratio Λ, regardless of its initial orientation angle θ_0, one can find the stable SS solution of (1) as

$$\zeta^p_{11,s} = \Lambda/G,\ \zeta^p_{22,s} = 1/(G\Lambda),\ \zeta^p_{12,s} = 0,\ \omega_s = \left[\sqrt{\Lambda\left(2G\Lambda^2 + \Lambda - 2G\right)} - 2G\Lambda\right]\Big/\left(1 + 2G\Lambda\right),\ \theta_s = 0 \qquad (2)$$

when 0<Λ<1 and for all possible values of *G*. This stable steady-state (SS) solution (2) corresponds to the configuration where the particle's initial major axis aligns with the extension direction.

The same SS result holds for three-dimensional ellipsoidal particles in a uniaxial extensional flow. The particle is found to align its initial major axis with the extension direction. Roscoe [1] predicted that for initially spherical, neo-Hookean particles, SS solutions may not be obtained when the particle stiffness is lower than a critical value. Due to the symmetry of the extensional flow, an initially spherical particle undergoes a pure stretching deformation, and deforms into spheroidal shapes. Plotting steady-state aspect ratio ω_s as a function of *G*, Fig. 2(a) shows that as *G* approaches a critical number $G^*\approx0.33$, the particle has a finite thickness ($\omega_s\approx0.15$). When *G* increases beyond G^*, the deformation of the particle never reaches steady state. Instead, the particle continues to stretch as ω reduces to zero, as shown in Fig. 2(b). This is because the elastic force inside the particle is insufficient to balance the viscous hydrodynamic force acting on the particle surface. The particle will keep stretching. The "phase diagram" for initially prolate spheroidal particles with the initial aspect ratios $\Lambda_1=\Lambda_2=\Lambda<1$ are shown in Fig. 2(c) as functions of both Λ and *G*.

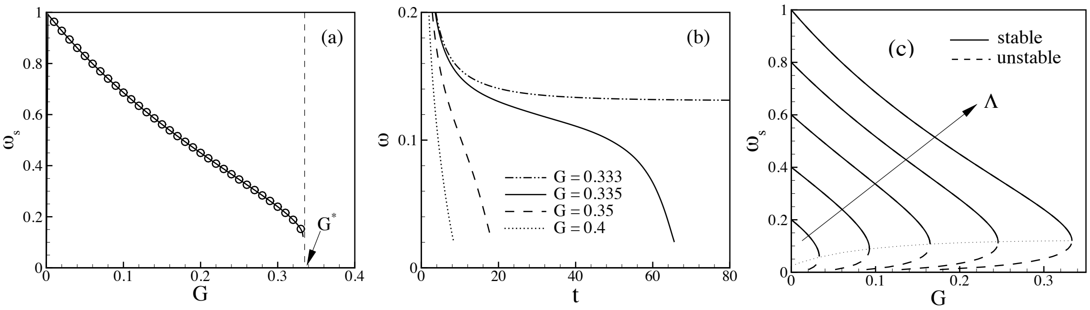

Fig 2. (a) SS solutions of ω_s as a function of *G* for initially spherical (Λ=1) particles. (b) Evolution of ω at $G>G^*\approx0.33$. (c) SS solutions of ω_s as a function of both Λ and *G*. Five $\omega_s(G)$ curves (from left to right) correspond to Λ= 0.2, 0.4, 0.6, 0.8, 1.0.

After solving the dynamics of a single particle, we can evaluate the rheology of a dilute suspension of monodispersed elastic particles in an extensional flow. To characterize the bulk property of the suspension in an extensional flow we introduce an "excess" extensional viscosity [1], $\mu_e = \left[\tau^p_{11} - \tau^p_{22} - 2(D_{11} - D_{22})\right]/\left[2(D_{11} - D_{22})\phi\right]$, where D_{11} and D_{22} are components of strain rate tensor and ϕ is the volume concentration of suspension. We found that the extensional viscosity of the suspension exhibits a general strain-thickening behavior, as shown in Fig. 3(a). For all values of Λ, the extensional viscosity approaches a maximum value as $G\to G^*$. For $G>G^*$, Fig. 3(b) shows that the bulk properties of the suspension become time-dependent: the extensional viscosity continues growing over time and eventually approaches infinity as the particle thickness reduces to zero. However, we must select the concentration ϕ carefully so that the assumption of a dilute suspension is valid.

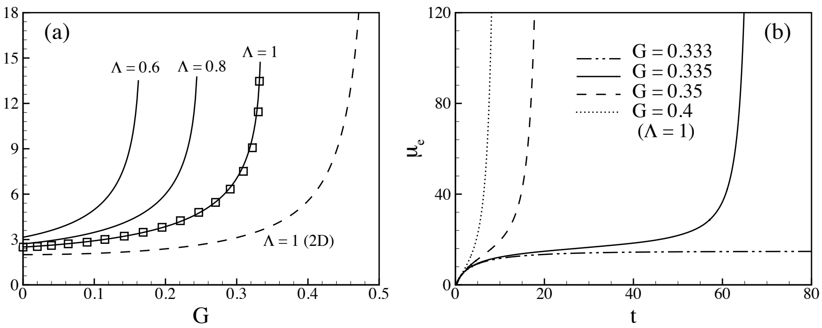

Fig 3. (a) SS values of the excess extensional viscosity μ_e of initially spheroidal particles when $G<G^*$. The dashed line represents the excess viscosity for a dilute suspension of initially circular, two-dimensional particles. (b) Time evolution of μ_e for a dilute suspension of initially spherical particles when $G>G^*$.

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