MODELING OF GRAVITATIONAL CONVECTION IN DISPERSE SYSTEMS: INSTABILITIES, VORTEX GENERATION, AND BOYCOTT EFFECT

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Summary A general two-fluid model of gravitational convection in two-phase disperse systems is proposed, similarity criteria controlling the convection are determined, and limiting cases of low-inertia particles and small and moderate Reynolds numbers of convection are studied numerically. Within 2D and quasi-2D approach for a narrow vertical enclosure, the development of vortex regions, instabilities of "fluid-suspension" interface, and the Boycott effect in settling suspension are investigated.

PROBLEM FORMULATION AND NUMERICAL RESULTS

The problem of gravitational convection in disperse systems is of practical importance due to a large number of natural and engineering applications employing gravity-driven phase separation (sedimentation, flotation, air-lift, hydraulic fracturing, etc.). Due to the development of computational technologies, in last years an interest increased in a quantitative description of two-phase gravitational convection and the explanation of some surprising effects, which where revealed experimentally but couldn't be interpreted on the basis of simple hydraulic or analytical models. One such effect is the increase in the effective sedimentation rate of a dispersed admixture in closed vessels, whose side walls are tilted from the gravity force direction. This effect was detected by a famous English scientist A. Boycott [1] when he observed sedimentation of erythrocytes, and since then, in the literature, it is called the Boycott effect. Enhanced sedimentation of an admixture in large vessels is associated with the onset of large-scale vortex zones in the settling suspension. The development of vortices is often accompanied by some mesoscale phenomena, such as strata formation in the dispersed phase and the development of instabilities on the interface between the suspension and pure fluid. The origins of the appearance of vortex zones and discontinuities in the dispersed-phase, and the influence of these factors on the effective sedimentation rate of suspensions in enclosures have not been well understood, that was the motivation for the present study.

We consider a closed volume filled with a viscous incompressible fluid containing non-deformable monodisperse spherical particles whose density differs from that of the fluid. Under the action of the gravity force the dispersed particles are set into motion, which induces gravitational convection and vortex formation in the carrier phase. We formulate a general two-fluid model of suspension, which in contrast to known models [2, 3] takes into account the effective "compressibility" of the medium describing the suspension as a whole, because the divergence of the mean-mass velocity of the suspension is non-zero due to a finite volume of the particles and a velocity slip between the phases. In general case, in the interphase momentum exchange the forces of nonstationary nature are also taken into account. The rheology of the medium is assumed to be Newtonian, with the effective viscosity dependent of the local particle volume fraction. In nondimensionalization, the characteristic scales are taken equal to the vessel height L and the particle settling velocity (for light-weight particles, the rise velocity) $U=mg|1-\eta|/6\pi\sigma\mu_0$ (m is the particle mass, σ is the particle radius, g is the gravity force acceleration, $\eta = \rho/\rho_s$ is the density ratio of the fluid and the particle material, μ_0 is the carrier-phase viscosity). The time scale is $T = L/U$. In nondimensional form and Cartesian coordinates, the equations of a general two-continuum model of two-phase gravitation convection read:

$$\mathrm{div}((1-c)\mathbf{v}+c\mathbf{v}_s)=0,\quad \frac{\partial c}{\partial t}+\mathrm{div}(c\mathbf{v}_s)=0,\quad \frac{d_s\mathbf{v}_s}{dt}=\mathbf{f}_s-\frac{\beta}{1-\eta}\mathbf{j},\quad \tau_{ij}=\left[\left(\frac{\partial v_j}{\partial x_i}+\frac{\partial v_i}{\partial x_j}\right)-\frac{2}{3}\delta_{ij}\mathrm{div}\mathbf{v}\right]$$

$$(1-c)\frac{d\mathbf{v}}{dt}+\frac{c}{\eta}\frac{d_s\mathbf{v}_s}{dt}=-\nabla p-\frac{\beta c}{1-\eta}\left(\frac{1}{\eta}-1\right)\mathbf{j}+\frac{1}{\mathrm{Re}}\sum_i\sum_j\frac{\partial}{\partial x_j}\varphi_0(c)\tau_{ij}\mathbf{k}_i \qquad (1)$$

$$\mathbf{f}_s=\beta\varphi_1(c)(\mathbf{v}-\mathbf{v}_s)+\frac{\eta}{2}\varphi_2(c)\left(\frac{d\mathbf{v}}{dt}-\frac{d_s\mathbf{v}_s}{dt}\right)+\eta\varphi_3(c)\left(\frac{d\mathbf{v}}{dt}+\frac{\beta}{1-\eta}\mathbf{j}\right)+\chi\beta\varphi_4(c)\int_0^t\frac{d_s(\mathbf{v}-\mathbf{v}_s)}{d\tau}\frac{d\tau}{\sqrt{t-\tau}}$$

$$\frac{d\mathbf{v}}{dt}=\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v},\quad \frac{d_s\mathbf{v}_s}{dt}=\frac{\partial\mathbf{v}_s}{\partial t}+(\mathbf{v}_s\cdot\nabla)\mathbf{v}_s,\qquad \beta=\frac{L}{l}=\frac{L(6\pi\sigma\mu_0)^2}{m^2g|1-\eta|},\qquad \chi=\sqrt{\frac{9\eta\beta}{2\pi}},\qquad \mathrm{Re}=\frac{\rho UL}{\mu_0}$$

Here, $\mathbf{v}$ and $\mathbf{v}_s$ are the velocities of the carrier phase and the particles, c is the particle volume fraction, $\mathbf{j}$ is the unit vector of the OY axis, directed opposite to the gravity force, l is the Stokes particle velocity relaxation length, δ_{ij} is the Kronecker delta, $\mathbf{k}_j$ is the notation for the basis vectors $\mathbf{k}_x = \mathbf{i}$, $\mathbf{k}_y = \mathbf{j}$, $\mathbf{k}_z = \mathbf{k}$, reduced pressure p signifies the quantity $(p^* - \rho g y^*)/\rho U^2$ (where necessary, the asterisk denotes dimensional variables). The interphase force $\mathbf{f}_s$ includes the Stokes drag, the added mass force, the Archimedes force, and the Basset force with certain correction functions φ_{1-4}, which should take into account the finiteness of particle volume fraction ($\varphi_{1-4} \to 1$ as $c \to 0$);. φ_0 is the Einstein correction to the effective viscosity. In this study, developing approach [4] we also constructed a set of simplified asymptotic models corresponding to typical real situations, which describe sedimentation of low inertia particles in large vessels, when the Reynolds numbers of convection are small and moderate. Calculations were performed in a 2D formulation and a quasi-2D formulation, constructed by averaging the three-dimensional equations over a narrow gap of a vertical slot. A finite difference method was proposed, based on using the "stream function - vorticity" variables for the carrier phase and "a flux limiter method" for the dispersed-phase continuity equation. Parametric calculations were performed, which made it possible to establish the origins of vortex generation in a settling suspension, resulting in the enhancement of particle sedimentation. Instabilities of the "suspension – pure fluid" interface and the formation of discontinuities in the dispersed-phase density field were studied both within 2D and quasi-2D models. On the basis of calculations, we demonstrated that the vortices are generated mostly in the regions of large horizontal gradients of particle concentration. For studying the Boycott effect, we calculated the gravitational convection developing in an initially uniform suspension, settling in a rectangular vessel with an impermeable diagonal wall (Fig. 1). In the calculations, the conditions approximately corresponded to the experiments [5]. Both in the experiments and in our calculations, the well distinct increase in the particle sedimentation velocity under the diagonal wall (Boycott effect) and the formation of horizontal strata in the particle density field were observed. The Boycott effect may be explained by the onset of an intense vortex, developing under the diagonal wall, so that in the appearing pure-fluid region the velocity is directed upward, while in the suspension it is directed downward (Fig. 1, right).

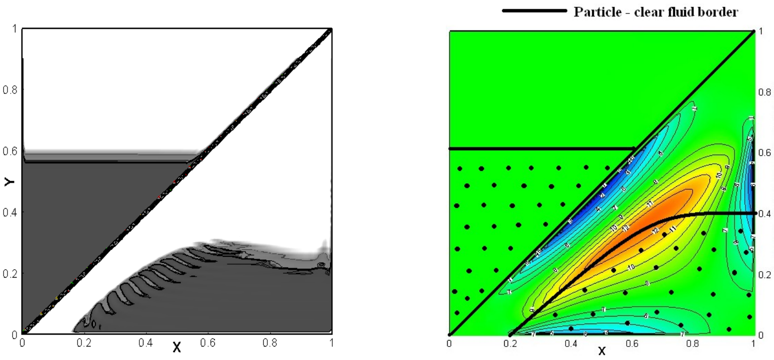

Fig.1. Typical distribution of sedimenting suspension in a vessel with a diagonal impermeable wall. Colored zones in the right figure show intense vortex regions.

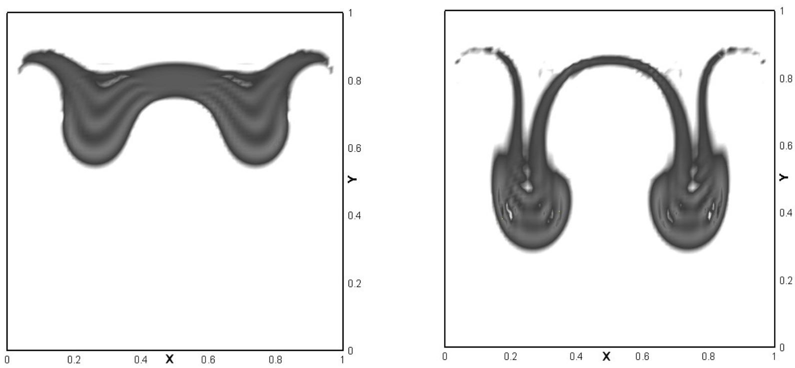

Fig. 2. Examples of instability development and strata formation in a sedimenting band-like region of suspension.

CONCLUSIONS

The model of two-phase gravitational convection proposed makes it possible to explain the mechanisms of vortex development and strata formation in settling suspension and to give quantitative estimates of the effect of enhanced sedimentation of suspensions in vessels with inclined boundaries. A qualitative comparison with known experimental data confirms the validity of the model.

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References

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