NUMERICAL INVESTIGATION OF LOCAL KOLMOGOROV SCALES IN TURBULENT RAYLEIGH-BÉNARD CONVECTION

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Summary Highly resolved direct numerical simulations (DNS) of turbulent Rayleigh-Bénard convection in a cylindrical container with aspect ratio unity and Prandtl number $Pr = 0.786$ have been performed for Rayleigh numbers *Ra* up to 10^9. The key ingredient for setting up a DNS is a sufficiently fine grid allowing to resolve the the local Kolmogorov scales. Usually this is done by fulfilling different published criteria for resolving the boundary layers and the bulk flow. To analyse if these criteria hold, absolute values of the smallest Kolmogorov scales obtained in DNS as well as their scaling with *Ra* are compared to the corresponding predictions from the theory. The analysis reveals that the smallest scales occurring in the boundary layer are about two thirds of the estimated ones. Further their scaling differs considerably from the above mentioned estimate while the bulk criterion fits quite well.

MOTIVATION & METHODOLOGY

Direct numerical simulation (DNS) is an important technique for the investigation of Rayleigh–Bénard convection (RBC). The key requirement to be fulfilled for an accurate DNS is thereby the spatial resolution of the computational grid. The grid in DNS consists mainly of two parts: a fine grid in the vicinity of the solid walls to resolve the thermal and viscous boundary layers (BLs) and a coarser grid in the bulk region apart from the wall. Both parts are to be analysed separately. As shown for example in [5], an insufficient resolution of the BLs can lead to large deviations even for integral quantities like the time- and spatially-averaged heat flux, namely the Nusselt number *Nu*. Therefore an analytical upper bound criterion for the necessary number of nodes within the BLs was derived in [4]. It is based on the assumption that the thermal BL thickness can be well approximated by $H/(2Nu)$ everywhere, where H is the height of the cell. A recent numerical study [6] pointed out that there are not only fluctuations in the BL region but also strongly varying thermal and viscous BL thicknesses (see the probability density functions in figure 1). Here the question arises, whether the assumption of a constant BL thickness $H/(2Nu)$ can lead to a proper estimate for the necessary resolution in the BL region.

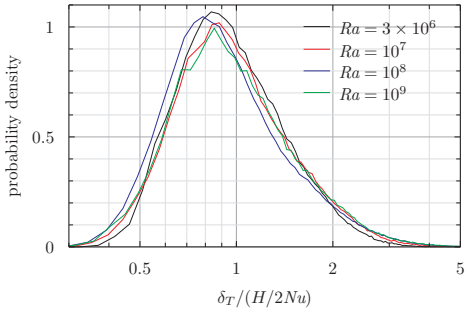

Figure 1. PDF of the thermal BL thickness δ_T in units of $H/2Nu$ at the heating plate obtained at a line along the large scale circulation and apart from the vertical wall, $Pr = 0.786,\ \Gamma = 1$.

Ra	N_r	N_ϕ	N_z	n_T	$\check{n}_T$	n_u	$\check{n}_u$
10^6	48	256	96	8	3	7	3
10^7	96	256	192	8	4	8	3
10^8	192	512	384	11	5	11	5
10^9	384	1024	768	13	7	12	6

Table 1. Simulation parameters for $Pr = 0.786, \Gamma = 1$: number of mesh nodes in i-direction N_i ($i = r, \phi, z$), number of nodes required to resolve global Kolmogorov scales N_k in vertical direction, number of nodes in the thermal/viscous BL as used in DNS (n_T/n_u) and as required by theory [4] ($\check{n}_T/\check{n}_u$).

Furthermore several estimates for the required global resolution exist [1],[4], which are based on the analytical relation for the volume- and time-averaged kinetic dissipation rate. For the global Kolmogorov scales η_K^{global} it is derived:

$$\eta_K^{global} = Pr^{1/2} Ra^{-1/4} (Nu - 1)^{-1/4} H \tag{1}$$

This estimate can be used as a criterion for the bulk resolution. But since the local kinetic dissipation rate varies strongly in space and is largest in the BL region [2], this requirement might be too strict. The motivation of the present work is to clarify the above issues.

For this purpose DNS of turbulent RBC in a cylindrical domain with aspect ratio $\Gamma = 1$ have been performed for $Ra = 10^6 - 10^9$ a Prandtl number $Pr = 0.786$. The latter corresponds to SF_6 at a pressure of 1 bar and about 20° C. The governing equations in Oberbeck-Boussinesq approximation are solved using a fourth order finite volume code for cylindrical coordinates [3]. Thereby the spatial resolution is chosen according to [4] but with an increased number of nodes in the BL and an increased resolution of the bulk. A list of simulation parameters is given in table 1.

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RESULTS

We analyse local Kolmogorov scales obtained from the local kinetic dissipation rates. Since of the main interest is their distribution in axial direction, we obtain profiles of the Kolmogorov scales by averaging in radial and azimuthal direction. This can be compared easily to the non-equidistant distribution of computational nodes in axial direction. Note that for the sake of low computational costs and high accuracy a structured grid is favourable in DNS, therefore the distribution of computational nodes in the vertical direction does not depend on the radial or azimuthal position. For $Ra = 10^8$ in figure 2 a comparison of the local Kolmogorov scale η_K^{local} with the estimates for the volume averaged (global) Kolmogorov scales η_K^{global} in eq. (1), the estimate from [4] for the Kolmogorov scales in the BL region η_K^{BL} (for $3 \times 10^{-4} \leq Pr \leq 1$),

$$\eta_K^{BL} = 2^{-3/2} a^{-1} Nu^{-3/2} Pr^{0.5355 - 0.033 \log_{10} Pr} H \quad \text{with} \;\; a = 0.482, \tag{2}$$

and the used grid spacing is depicted.

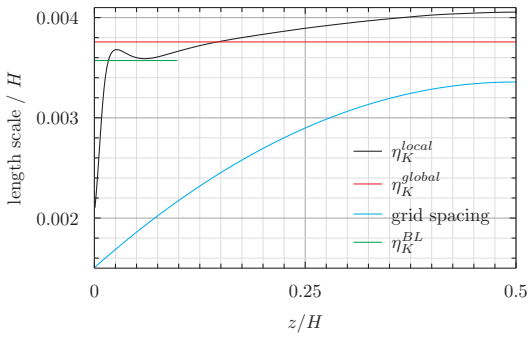

Figure 2. Axial profile of the local Kolmogorov scales (η_K^{local}) in comparison with the global estimate η_K^{global} from eq. (1), the estimate for the BL region η_K^{BL} from eq. (2) and the used grid spacing for $Ra = 10^8,\ Pr = 0.786,\ \Gamma = 1$.

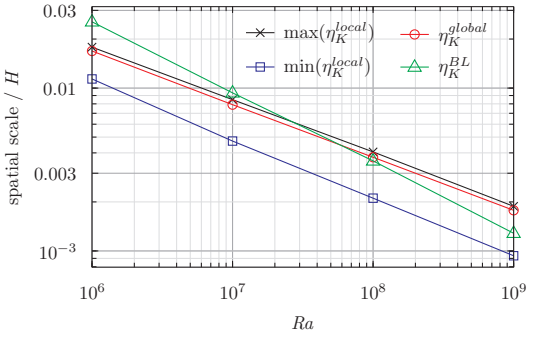

Figure 3. Scaling of the minimum and maximum of the local Kolmogorov scale η_K^{local} with Ra in comparison with estimates for the global Kolmogorov scale η_K^{global} from eq. (1) and the estimate for the Kolmogorov scale in the boundary layer η_K^{BL} from eq. (2) for $Pr = 0.786,\ \Gamma = 1$.

Figure 2 reveals that the Kolmogorov scales are everywhere larger than the used grid spacing as demanded for a DNS. Furthermore it is shown in figure 2 and 3 that the estimated bulk resolution is slightly smaller than the maximum of the local Kolmogorov scales and decreases with Ra just like η_K^{global} (for $10^6 \leq Ra \leq 10^9$). Thus we conclude that eq. (1) estimates the bulk resolution well which was a priori not guaranteed since η_K^{local} varies strongly in space.
The analysis of figures 2 and 3 reveals that the estimated value η_K^{BL} overpredicts the minimum of η_K^{local} near the horizontal walls. Furthermore the scaling of the minimum with Ra in figure 3 differs from the criterion in eq. (2). The reasons for this deviations might be that the parameter $a = 0.482$ in eq. (2) is a result of the fitting of experimental data. In contrast using the DNS data from [6] $a \approx 1.5$ is obtained. In addition the ratio of thermal and viscous BL thicknesses is assumed to be constant for deriving eq. (2), while in [6] a weak Ra-dependence is found. This might explain the difference in the scaling with Ra. From the above analysis we conclude that the estimate in equation (2) is not sufficient for resolving the smallest Kolmogorov scales in the boundary layer region. For the considered $Ra,\ Pr,\ \Gamma$ it seems reasonable for us to use rather $\eta_K^{BL}/2$. Nevertheless further investigations are necessary to obtain a proper estimate for other Ra and Pr.

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