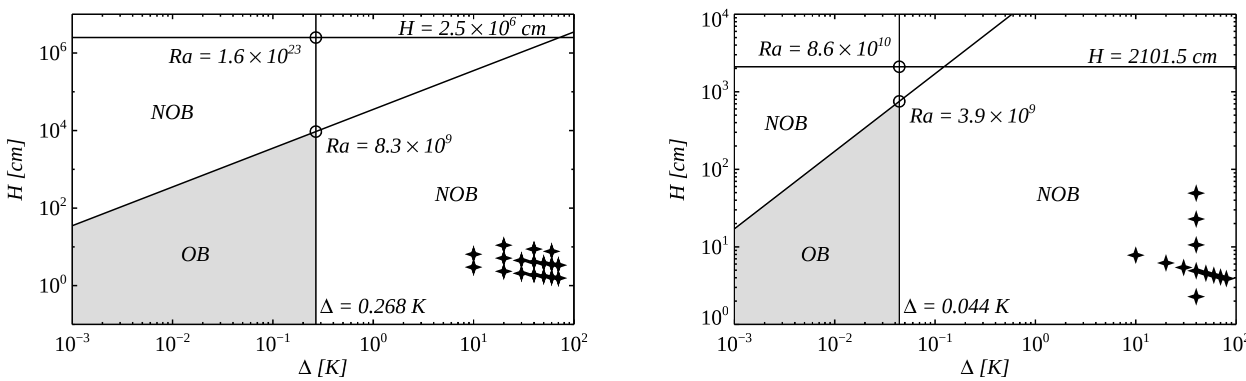
RAYLEIGH-BÉNARD CONVECTION IN LIQUIDS WITH TEMPERATURE-DEPENDENT MATERIAL PROPERTIES

Susanne Horn*[a)], Olga Shishkina* & Claus Wagner*
*Institute of Aerodynamics and Flow Technology, German Aerospace Center (DLR), 37073 Göttingen, Germany

Summary The influence of non-Oberbeck-Boussinesq (NOB) effects on Rayleigh-Bénard convection in liquids is investigated by means of three-dimensional direct numerical simulations (DNS). For this purpose the impact of temperature dependent material properties is studied in water with a Prandtl number of $Pr = 4.38$ within a range of Rayleigh numbers of $10^7 \leq Ra \leq 10^9$ and in glycerol with $Pr = 2547.9$ and within $10^5 \leq Ra \leq 10^9$. The imposed temperature differences Δ reach up to $70\,\mathrm{K}$ in the case of water and up to $80\,\mathrm{K}$ in the case of glycerol. The generated flow fields are analysed with respect to deviations from the Oberbeck-Boussinesq (OB) case. We obtain a breakdown of the top-bottom symmetry, that is, different boundary layer thicknesses, modified mean temperature profiles, entailing an increase of the centre temperature, and asymmetric velocity flow patterns. Furthermore the influence on the scaling behaviour of the Reynolds number Re and the Nusselt number Nu is studied.

INTRODUCTION

Rayleigh-Bénard convection is one of the basic paradigms to study the in general highly complex buoyancy driven flows occurring in nature as well as in engineering. This system is simply a fluid confined between a cooling plate at the top and a heating plate at the bottom. However, the theoretical treatment is less simple. Thus, commonly the Oberbeck-Boussinesq (OB) approximation is employed, meaning that the fluid is incompressible and all material properties are assumed to be constant, except the density ρ within the buoyancy part, that varies linearly with the temperature T. Gray & Giorgini [2] developed a mathematically straightforward way to calculate the validity range of the OB approximation, proving that this approach is indeed quite restrictive. We show the resulting validity range for water and glycerol in figure 1, and in addition also the simulations conducted.



(a) Water ($Pr = 4.38$) (b) Glycerol ($Pr = 2547.9$)

Figure 1: Region of validity of the Oberbeck–Boussinesq approximation for (a) water and (b) glycerol at $T_m = 40\,^\circ\mathrm{C}$, according to Gray & Giorgini [2]. The grey shaded areas show the parameter range where the OB assumption is applicable. Our NOB DNS data points are denoted by stars.

NUMERICAL METHODOLOGY

For our studies of Rayleigh-Bénard convection we apply a fourth order finite volume method, developed by Shishkina & Wagner [4]. For the purpose of investigating non-Oberbeck-Boussinesq (NOB), the code was advanced by implementing temperature dependent material properties. The viscosity ν, the heat conductivity Λ, the diffusivity κ and the density ρ in the buoyancy term are described by polynomials up to cubic order in the case of water and up to septic order in the case of glycerol, where the respective coefficients are adopted from Ahlers et al. [1] and Segur & Oberstar [3]. The density elsewhere and the isobaric specific heat capacity c_p are set constant to their values at the arithmetic mean temperature T_m. This approach is appropriate for almost all liquids and accounts for the major NOB effects.

a) Corresponding author. E-mail: susanne.horn@dlr.de

As control parameters of our simulations serve the imposed temperature difference Δ, as well as the Rayleigh and the Prandtl number at the mean temperature, set to $T_m = 40\,^\circ\mathrm{C}$,

$$Ra = \frac{\alpha_m g \Delta T H^3}{\kappa_m \nu_m}, \qquad \mathrm{Pr} = \frac{\nu_m}{\kappa_m}.$$

All simulations are performed in a cylindrical domain with an aspect ratio of $\Gamma = D/H = 1$, where D is the diameter and H the height of the Rayleigh-Bénard cell.

RESULTS AND OUTLOOK

Currently, we are conducting simulations of Rayleigh-Bénard convection in glycerol and water under OB and various NOB conditions, that is different temperature differences Δ ranging from $10\,\mathrm{K}$ to $80\,\mathrm{K}$ and Rayleigh numbers ranging from 10^5 to 10^9. Thus, we will provide an in-depth analysis of a wide parameter space in Ra and Δ.

In figure 2 instantaneous flow fields of the two investigated fluids water and glycerol are presented. They do not only qualitatively show the very different behaviour in a moderate and a high Prandtl number fluid, but also the most distinct NOB effects. We find an increase of the centre temperature in the bulk and an asymmetric plume behaviour due to the varying viscosity, i.e. the warm plumes from the bottom layer move faster upwards compared to the OB case while, vice versa, the cold plumes from the top layer move downwards more slowly. Moreover, we find that the top boundary layers are always thicker than the bottom ones in both fluids. Additionally, in glycerol we also observe different coherent flow structures developing under the different NOB conditions.

(a) Water ($Ra = 10^8$, OB) (b) Water ($Ra = 10^8$, NOB) (c) Glycerol ($Ra = 10^9$, OB) (d) Glycerol ($Ra = 10^9$, NOB)

Figure 2: Flow fields for $Pr = 4.38$ (water) and $Pr = 2547.9$ (glycerol) under OB and NOB ($\Delta = 40\,\mathrm{K}$) conditions.

In the final contribution we will draw quantitative conclusions and present detailed statistics, such as probability density functions and high order moments, the deviation of the centre temperature T_c from the mean temperature T_m with Δ and the influence on the scaling behaviour of the Reynolds number Re and the Nusselt number Nu will be discussed. Finally, our three dimensional DNS data will be validated against experimental data, e.g. from Ahlers et al. [1], and 2-D simulations by Sugiyama et al. [6, 7] and compared to the 3-D OB simulations in water by Shishkina & Thess [5].

Acknowledgment

The authors acknowledge support by the *Deutsche Forschungsgemeinschaft (DFG)* under grant SH405/2-1. Furthermore, the authors would also like to thank the *Leibniz-Rechenzentrum (LRZ)* in Garching for providing computational resources.

References

[1] Ahlers G., Brown E., Fontenele Araujo F., Funfschilling D., Grossmann S., Lohse D.:Non-Oberbeck-Boussinesq effects in strongly turbulent Rayleigh-Bénard convection. *J. Fluid Mech.* **569**:409–445, 2006.

[2] Gray D.D., Giorgini A.: The validity of the Boussinesq approximation for liquids and gases. *Int. J. Heat Mass Transfer* **19**:545–551, 1976

[3] Segur J.B., Oberstar H.E.: Viscosity of Glycerol and Its Aqueous Solutions. *Ind. Eng. Chem.* **43**:2117–2120, 1951

[4] Shishkina O., Wagner C.: A fourth order accurate finite volume scheme for numerical simulations of turbulent Rayleigh-Bénard convection in cylindrical containers. *C. R. Mecanique* **333**:17–28, 2005

[5] Shishkina O., Thess A.: Mean temperature profiles in turbulent Rayleigh-Bénard convection of water. *J. Fluid Mech.* **633**:449–460, 2009

[6] Sugiyama K., Calzavarini E., Grossmann S., Lohse D.: Non-Oberbeck-Boussinesq effects in two-dimensional Rayleigh-Bénard convection in glycerol. *Europhys. Lett.***80**:34002, 2007

[7] Sugiyama K., Calzavarini E., Grossmann S., Lohse D.: Flow organization in two-dimensional non-Oberbeck-Boussinesq Rayleigh-Bénard convection in water. *J. Fluid Mech.* **637**:105–135, 2009