ANALYSING THE SPATIO-TEMPORAL FLOW BEHAVIOR IN RB-CONVECTION USING GLOBAL STATISTICAL ANALYSIS

U. Hansen[a)] & K. Petschel
Institut für Geophysik, Westfälische-Wilhelms-Universität Münster, Germany

Summary The analysis of long term data sets obtained by numerical simulations of turbulent 2D Rayleigh-Bénard convection allows us to get a more comprehensive view of the spatio-temporal flow behavior. By means of a global statistical analysis of the characteristic spatial modes of the flow we extract information about the stability of dominant large-scale modes as well as the reversal paths in state sub-space. This provides information about the coexistence of dominant modes.

INTRODUCTION

Turbulent thermal convection is a physical process which can be observed in many natural systems such as the atmosphere, the oceans and the Earth's interior. Thus, many recent efforts focused on Rayleigh-Bénard convection, .i.e. a buoyancy driven flow within a fluid layer heated from below. Even though thermal convection is a classical and intensively studied hydrodynamical problem, it still comprises several open questions (see a recent review paper by Ahlers et al. [1]). The past years saw active research in the field where many efforts centered around irregular reversals in the orientation of the large-scale circulation (LSC) observed in several experimental (e.g., [7, 14, 5]) and numerical studies (e.g., [10, 4, 13]). Such reversals in convecting systems can also be observed in nature. Prominent examples are the reversing geodynamo and reversals in the wind direction of the atmosphere. However most studies examine turbulent thermal convection applying rigid boundary conditions, and therefore several phenomena are observed related to viscous boundary layers [8, 9]. For example, reversals of LSC are often characterized by the appearance of corner flows, i.e., counterrotating flow structures generated by the viscous boundary layer [13]. Though stress-free boundary conditions are experimentally difficult to realize, they are a good approach for many systems in nature. Many examples can be found in geophysics or astrophysics, such as convection in stars, in the ocean, or in the earth's mantle.
In contrast to laboratory studies, direct numerical simulations (DNSs) provide a comprehensive picture of the spatial field structure and are therefore very suitable for examining structural reorganizations within convective flows. The DNS, however, is limited in its spatio-temporal resolution due to computational restrictions.
We were originally motivated by studying convective flows within planetary mantles. Due to the high viscosity of mantle rocks, the value of the Prandtl number given by $\mathrm{Pr} = \nu/\kappa$ (ν,κ denote kinematic viscosity and thermal diffusivity, respectively) is estimated to be around 10^{23} [11]. In this case it is reasonable to consider the infinite Prandtl number limit, implying that mechanical inertia is neglected in the momentum equation. Intrinsically, infinite Prandtl number convection does not contain toroidal motion (horizontal vortices).

LARGE-SCALE CIRCULATION

Breuer and Hansen [4] have shown that even in the zero Reynolds number limit thermal convection exhibits many features which are commonly addressed to turbulent flows. Certainly, the dynamical behavior of high Reynolds number flows differs from the infinite Prandtl case in some sense. However we find it remarkable that thermal convection dynamically lacking explicit time dependence and mechanical inertia shows many properties of turbulence which are hardly distinguishable from convective flows at high Reynolds numbers. Special attention has been given by Petschel et al.[10] to the phenomenon of reversals in the orientation of the underlying large-scale circulation of the flow. The convective flow of our long term simulation at $\mathrm{Ra} = 10^8$ and aspect ratio $\Gamma = 2$ features a developed LSC superimposed by small-scale boundary layer instabilities in form of thin plume-like up- and down-wellings. These boundary layer instabilities are irregular, disconnected structures carried by the thermal wind. The LSC turns out to be only temporarily stable by switching predominantly between clockwise or anti- clockwise orientated one- and two-cell patterns, respectively.

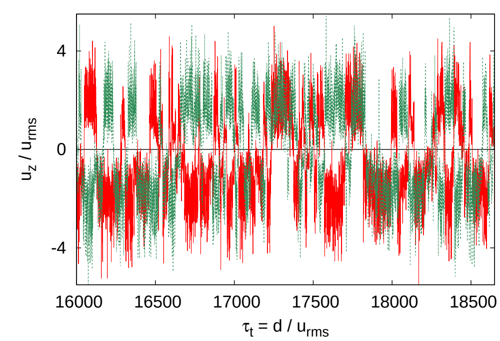

Figure 1. Temporal evolution of the vertical velocity at the side-walls (solid red line left and dashed green line right) at mid-depth indicating reversals in the orientation of the large-scale circulation. If the velocity components have different signs, the system must be organized into an odd number of convection cells, and if the signs are the same the flow is organized into an even number of convection cells.

As stated above, a characteristic feature of high Rayleigh number convection appears to be a frequent reorientation of the

a) Corresponding author. E-mail: hansen@earth.uni-muenster.de

LSC. This is illustrated in figure 1 showing the temporal evolution of the vertical velocity at two distinct measurement points, located at mid-depth at the left and the right side-wall, respectively. One can clearly see the temporal correlation of these two measurements reflecting the large-scale dynamics of the system. A change in sign in both measurements at the side-walls indicates a global reorientation of the LSC also termed flow reversal. Concerning this matter, the prominent question is: What triggers such a reversal and what are the characteristics of a reversal path?

PRIMARY MODE DECOMPOSITION

Breuer and Hansen [4] showed for similar model parameters that there exist four metastable states that turned out to be stationary solutions of the governing equations. This indicates that flow reversals are an intrinsic feature of turbulent thermal convection in the form of competing states whose main modes turn out to be solutions of the fix point equations. The temporal flow structure can now be interpreted by different primary modes superimposed by plumes that act in a sense as noise. To analyze the large-scale dynamics, i.e., flow reversals and primary mode selection, it is reasonable to separate the large-scale wind pattern from the small-scale plume dynamics. Due to statistical analysis of the eigenfunctions of the Laplacian $\Delta \mathbf{u}$ in the momentum equation, we are able to extract the dependency between the predominant modes as well as the reversal path in state subspace of these modes.

CONCLUSIONS

Our study indicates, unlike previous work [2, 12, 3, 6], that a generic mechanism always involves transitions over higher-order modes. Such a reversal goes along with a breakdown of the LSC through a strong interaction of plumes with the bulk, an excitation of higher flow modes, a breakdown of this partly stable flow pattern, and a reestablishment of a reversed LSC. Thus the study of Petschel et al.[10] allows us to characterize the dynamics and the irregular reorientation of the LSC.

References

[1] G. Ahlers, S. Grossmann, and D. Lohse. Heat transfer and large scale dynamics in turbulent Rayleigh-Bénard convection. *Rev. Mod. Phys.*, 81:503–537, 2009.

[2] F. A. Araujo, S. Grossmann, and D. Lohse. Wind reversals in turbulent Rayleigh-Bénard convection. *Phys. Rev. Lett.*, 95:084502, 2005.

[3] R. Benzi. Flow reversals in a simple dynamical model of turbulence. *Phys. Rev. Lett.*, 95:024502, 2005.

[4] M. Breuer and U. Hansen. Turbulent convection in the zero reynolds number limit. *Europhys. Lett.*, 86:24004, 2009.

[5] E. Brown and G. Ahlers. Rotations and cessations of the large-scale circulation in turbulent rayleigh beénard convection. *J. Fluid Mech.*, 568:351–386, 2006.

[6] E. Brown and G. Ahlers. A model of diffusion in a potential well for the dynamics of the large-scale circulation in turbulent rayleigh-bé convection. *Phys. Fluids*, 20:075101, 2008.

[7] S. Cioni, S. Ciliberto, and J. Sommeria. Strongly turbulent Rayleigh-Bénard convection in mercury: comparison with results at moderate Prandtl number. *J. Fluid Mech.*, 335:111–140, 1997.

[8] S. Grossmann and D. Lohse. Scaling in thermal convection: a unifying theory. *J. Fluid Mech.*, 407:27–56, 2000.

[9] S. Grossmann and D. Lohse. Thermal convection for large Prandtl numbers. *Phys. Rev. Lett.*, 86:3316–3319, 2001.

[10] K. Petschel, M. Wilczek, M. Breuer, R. Friedrich, and U. Hansen. Statistical analysis of global wind dynamics in vigorous rayleigh-bénard convection. *Phys. Rev E*, 84:026309, 2011.

[11] G. Schubert, D. L. Turcotte, and P. Olson. *Mantle Convection in the Earth and Planets*. Cambridge University Press, 2001.

[12] K. R. Sreenivasan, A. Bershadskii, and J. J. Niemela. Mean wind and its reversals in thermal convection. *Phys. Rev. E*, 65:056306, 2002.

[13] K. Sugiyama, R. Ni, R. J. A. M. Stevens, T. S. Chan, S. Q. Zhou, H. D. Xi, C. Sun, S. Grossmann, K. Q. Xia, and D. Lohse. Flow reversals in thermally driven turbulence. *Phys. Rev. Lett*, 105:034503, 2010.

[14] H. D. Xi and K.-Q. Xia. Azimuthal motion, reorientation, cessation, and reversal of the large-scale circulation in turbulent thermal convection: A comparative study in aspect ratio one and one-half geometries. *Phys. Rev. E*, 78:036326–1–11, 2008.