# STABILITY OF A NATURAL CONVECTION FLOW INSIDE A CYLINDRICAL CAVITY PARTIALLY HEATED ON THE SIDE

José Núñez*, Eduardo Ramos* a) & Juan M. Lopez**
* *Center for Energy Research, Universidad Nacional Autónoma de México, 62580, Temixco, Mor. Mexico*
** *School of Mathematical and Statistical Sciences, Arizona State University, Tempe AZ, USA*

*Summary* Three-dimensional flows are simulated in a vertical cylinder partially heated from the side. The vertical wall is considered to be at a high temperature up to 3/4 of its total height while the rest of the vertical wall is thermally insulated. The top and bottom walls are kept at cold and hot temperatures respectively. The cylinder aspect ratio A = height/ diameter is 1.25, whereas a fixed Prandtl number, Pr = 6.5, is considered. A hybrid finite volume–spectral numerical method is used to integrate the governing equations. With this method, an azimuthal mode decomposition analysis to characterize the flows and their transitions can be naturally accomplished. The base flow obtained for small, non zero Rayleigh number is a constant velocity, axisymmetric motion which loses stability to a $k$=1 azimuthal mode flow at approximately Ra = $1.7 \times 10^4$. The most vigorous motion is observed at the upper region of the cavity where the vertical temperature gradient is large, while the lower part of the cavity, the flow is practically stagnant and at a constant temperature. Further increase of the Rayleigh number leads to the occurrence of time dependent flow patterns with motion in the whole cavity. In the range $5 \times 10^5 < Ra < 1 \times 10^6$, flows with azimuthal modes $k$ = 3, 4 and 5 were found.

## INTRODUCTION

It is now accepted that the dynamics of the molten material in crucibles determines the optical and electronic properties of the crystals grown using Bridgman or Czochralski methods. Reviews on these phenomena can be found in references [1] and [2]. The natural convective flow in containers has been studied considering several configurations. The classical arrangement of bottom and top walls kept at high and low temperatures has been extensively studied (see for instance [6]), but the flow in a vertical cylinder partially heated on the side wall has not been much analyzed. When the cylindrical container is kept at a high temperature at a central portion of the vertical wall and the top and bottom walls are kept at a low temperature (Bridgman technique) has been described [3], [4] [5] and several transitions to non-axisymmetric, steady or oscillatory states have been reported.

## PROBLEM FORMULATION

We describe the flow patterns and present a stability analysis of a natural convective flow inside a vertical cylinder with the bottom wall and a section of the lateral wall kept at a high temperature and the top wall at a low temperature. These conditions are shown in Figure 1. The boundary conditions considered lead to an unstable situation in the upper region of

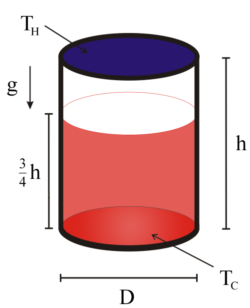


**Figure 1.** Geometry of the problem and temperature boundary conditions. The regions in red and blue are kept at a high ($T_H$) and low ($T_C$) temperatures respectively. The rest of the container is thermally insulated.

the container with hot fluid at the bottom and cold at the top. This situation is similar to the upper part of the Bridgman configuration. The qualitative behavior of the flow depends on three parameters, the Rayleigh number, the Prandtl number and the aspect ratio of the cylindrical container. The aspect ratio of the cylindrical container (height/diameter) is 1.25 and the Prandtl number is fixed at 6.7 which corresponds approximately to water at room temperature. The analysis is based on the the solution of the mass, momentum, and energy conservation equations with the Boussinesq approximation. The boundary conditions considered represent the physical situation described in Figure 1. The method used to solve the conservation equations is based on the combined use of Fourier Galerkin and finite volume techniques. A Fourier expansion is made in the angular direction, partially translating the problem to the Fourier space and then solve the resulting equations using a finite volume technique. This method is described in detail in [7].

a) Corresponding author. E-mail: erm@cie.unam.mx.

## RESULTS AND DISCUSSION

For Rayleigh numbers smaller than $Ra = 10^4$ , the convective pattern is an axisymmetric convective cell with small radial and axial velocities and zero azimuthal velocity. The vortex core is toroidal and is confined to the upper part of the container; the rest of the fluid remains practically motionless. At approximately $Ra = 1.7 \times 10^4$ a symmetry braking bifurcation occurs with azimuthal wave number $k = 1$. The growth of the kinetic energy of the azimuthal velocity as a function of time, starting with an initial perturbation of $10^{-7}$ is shown in the left side of Figure 2 for $Ra = 2 \times 10^4$. The flow is time independent and is mostly confined to the upper region of the container. The distribution of axial velocity at $z = 0.9\ h$ is shown on the right of Figure 2.

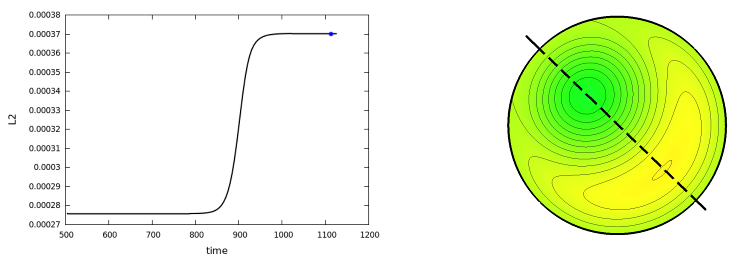


**Figure 2.** Left: Total kinetic energy ($L^2$– norm) as a function of time. Right: Contours of axial velocity in the cross section of the cavity at $z = 0.9\ h$ for time = 450 (blue dot on the graph on the left). In the two plots, $Ra = 2 \times 10^4$.

At higher Rayleigh numbers, the steady state solution looses stability and the flow becomes time dependent. For $Ra = 5 \times 10^5$, we find that the flow pattern changes irregularly for the time interval studied. At certain times, the solution in the upper section of the cavity the flow is dominated by the $k = 3$ or $k = 4$ azimuthal wave numbers. This dynamic behavior is exemplified in Figure 3.

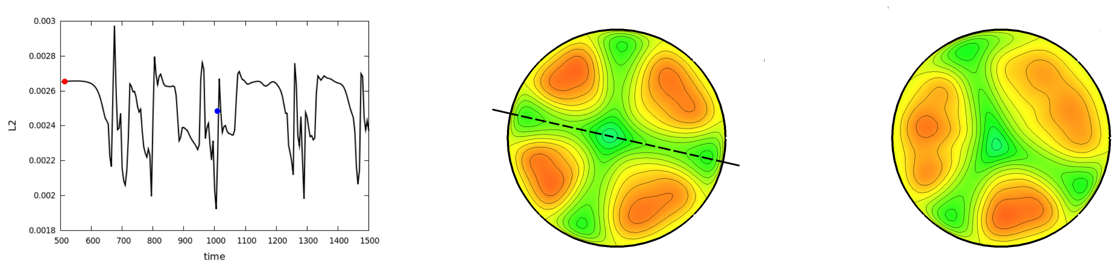


**Figure 3.** Left: Total kinetic energy ($L^2$– norm) as a function of time. Center: Contours of axial velocity in the cross section of the cavity at $z = 0.9\ h$ for time = 505 (red dot on the graph on the left). Right: Contours of axial velocity in the cross section of the cavity at $z = 0.9\ h$ for time = 1020 (blue dot on the graph on the left). In all plots, $Ra = 5 \times 10^5$.

## CONCLUSIONS

The natural convective flow in a partially heated cylindrical container has been analyzed from the topological and stability points of view. Various flow patterns have been found and the conditions required for their occurrence have been identified.

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