CONJUGATED CONVECTIVE-CONDUCTIVE HEAT TRANSFER IN MICRO CHANNELS ON POLYMERIC NANOCOMPOSITE SUBSTRATE

Diego C. Knupp, Carolina P. Naveira Cotta & Renato M. Cotta [a)]
Universidade Federal do Rio de Janeiro - COPPE & POLI, UFRJ
Mechanical Engineering Department - Laboratory of Transmission and Technology of Heat
Cx. Postal 68503, Rio de Janeiro, RJ, 21945-970, Brazil

Summary This work analyzes conjugated heat transfer in rectangular micro-channels etched on polymeric nanocomposite substrates, employing a single domain formulation for modelling the heat transfer phenomena at both the fluid stream and the channel walls regions. By making use of coefficients represented as space variable functions, with abrupt transitions occurring at the fluid-wall interface, the mathematical model is fed with the information concerning the transition of the two domains, unifying the model into a single domain formulation. The Generalized Integral Transform Technique (GITT) [1] is then employed in the hybrid numerical-analytical solution of the resulting convection-diffusion problem with variable coefficients. An actual configuration is considered consisting of a micro-channel manufactured on a nanocomposite made of polyester resin and alumina nanoparticles and the effects of axial conduction in both the wall and fluid regions is investigated, besides considering the effect of the adiabatic region upstream of the heat exchange section.

PROBLEM FORMULATION AND SOLUTION METHODOLOGY

The problem involves a laminar incompressible internal flow of a Newtonian fluid between parallel plates in steady-state, undergoing conjugate heat transfer. The external face of the micro-channel wall exchanges heat with the surrounding environment by convection (Fig.1). The channel wall is considered to participate on the heat transfer problem through both transversal and axial heat conduction. The fluid flows with a known fully developed velocity profile $u_f(y)$, and with uniform inlet temperature T_{in}.

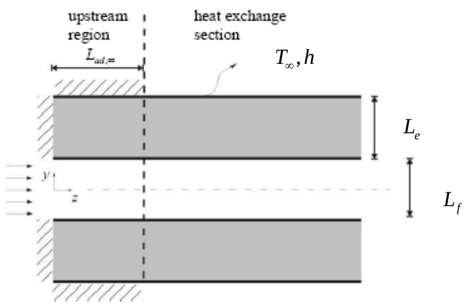


Figure 1 – Schematic representation of the conjugate heat transfer problem in the micro-channel

The flow is hydrodynamically developed and thermally developing. The formulation of the conjugate problem as a single region model is achieved by making use of space variable coefficients with abrupt transitions at the fluid-solid wall interface, in the form:

$$u(y)w_f \frac{\partial T(y,z)}{\partial z} = k(y)\frac{\partial^2 T}{\partial z^2} + \frac{\partial}{\partial y}\left(k(y)\frac{\partial T}{\partial y}\right), \; 0 < y < L_e, \; z > 0 \tag{1a}$$

$$\begin{cases} T(y, z=0) = T_{in}, & 0 \le y \le L_f \\ \left.\dfrac{\partial T}{\partial z}\right|_{z=0} = 0, & L_f \le y \le L_e \end{cases} \qquad \left.\frac{\partial T}{\partial y}\right|_{y=0} = 0, \quad \left.\frac{\partial T}{\partial y}\right|_{y=L_e} + T(y=L_e,\, z) = T_\infty \tag{1b-d}$$

$$u(y) = \begin{cases} u_f(y), & \text{if } 0 < y < L_f \\ 0, & \text{if } L_f < y < L_e \end{cases} \qquad k(y) = \begin{cases} k_f, & \text{if } 0 < y < L_f \\ k_s, & \text{if } L_f < y < L_e \end{cases} \tag{1e,f}$$

where w_f is the heat capacity of the fluid, k_s is the thermal conductivity of the channel wall, k_f is the thermal conductivity of the fluid, and $u_f(y)$ is the parabolic velocity profile. The following dimensionless groups are defined:

$$Z = \frac{z/D_h}{\mathrm{Re}\,\mathrm{Pr}} = \frac{z}{D_h \mathrm{Pe}}; \; Y = \frac{y}{L_e}; \; U = \frac{u}{u_{av}}; \theta = \frac{T - T_\infty}{T_{in} - T_\infty}; \; K = \frac{k}{k_f}; \; \mathrm{Re} = \frac{u_{av} D_h}{\nu};$$
$$\mathrm{Pr} = \frac{\nu}{\alpha}; \; \mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr} = \frac{u_{av} D_h}{\alpha}; \; \alpha = \frac{k_f}{w_f} \tag{2a-i}$$

where the hydraulic diameter is given by $D_h = 2L_f$ since $L_w >> L_f$, L_w and L_f are the micro-channel width and height, respectively. To solve problem (1) we make use of a partial integral transformation scheme of the GITT [2] with the inclusion of a pseudo-transient term. After making use of eqs. (2a-i), one obtains the dimensionless formulation:

a) Corresponding author. Email: cotta@mecanica.coppe.ufrj.br

$$\frac{\partial\theta\ (Y,Z,t)}{\partial t}=-\frac{U(Y)}{4}\frac{\partial\theta}{\partial Z}+\frac{K(Y)}{(2\mathrm{Pe})^2}\frac{\partial^2\theta}{\partial Z^2}+\frac{\partial}{\partial Y}\left(K(Y)\frac{\partial\theta}{\partial Y}\right),0<Y<1,\ \ 0<Z<Z_\infty,\quad t>0 \tag{3a}$$

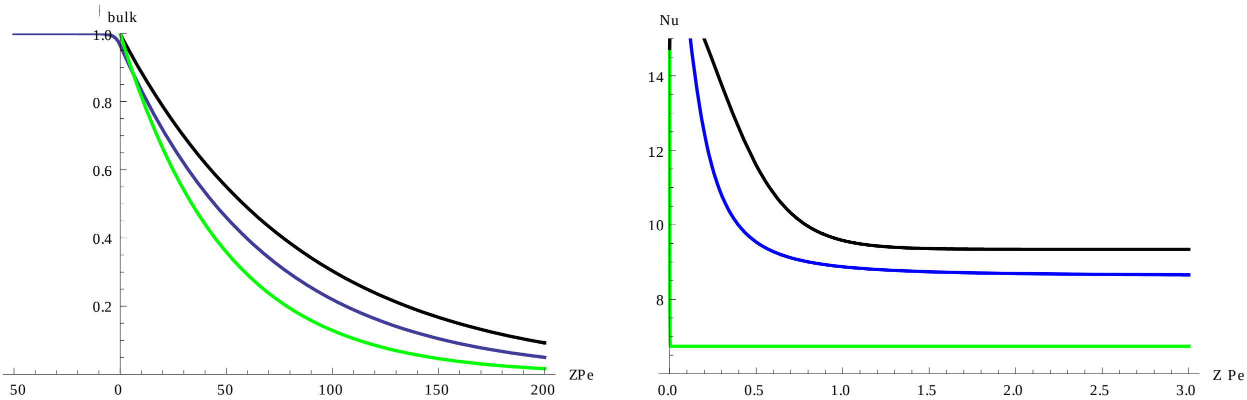
$$\theta\ (t=0)=\theta_{t=0},\quad 0<Y<1,\ 0<Z<Z_\infty,\quad \left.\frac{\partial\theta}{\partial Y}\right|_{Y=0}=0,\quad \left.\frac{\partial\theta}{\partial Y}\right|_{Y=1}+\mathrm{Bi}\ \theta\ (Y=1)=0,\quad 0<Z<Z_\infty,\quad t>0 \tag{3b-d}$$

$$\theta_{Z=0}=1-\frac{1}{\lambda(Y)}\left.\frac{d\theta}{dZ}\right|_{Z=0},\quad \left.\frac{d\theta}{dZ}\right|_{Z=Z_\infty}=0,\ 0<Y<1,\ t>0 \tag{3e,f}$$

where $\lambda(Y)$ is a step function that tends to infinity for $0\le Y\le L_f/L_e$ and tends to zero for $L_f/L_e\le Y\le 1$. The initial condition $\theta_{t=0}$ is preferably an estimate of the steady-state solution, since we aim the steady state results. When the upstream region is taken into account, a new problem is formulated for that region by simply substituting the convective boundary condition at the external wall, in eq. (3d), by an adiabatic boundary. Then the problem is solved by coupling the adiabatic upstream region ($-L_{ad,\infty}\le Z\le 0$) and the heat exchange section ($0\le Z\le Z_\infty$).

RESULTS AND DISCUSSION

The dimensionless thermophysical properties have been calculated motivated by an application with a micro-channel manufactured on a nanocomposite material [3], made of polyester resin filled with aluminum oxide nanoparticles to obtain an enhanced thermal conductivity ($k_s=0.204$ W/m°C) with water as the working fluid ($k_f=0.64$ W/m°C), so that $k_s/k_f=0.32$. Considering a rectangular micro-channel with height $L_f=200\mu m$, width $L_w=4cm$ and wall thickness $L_e=400\mu m$ and a prescribed volumetric flow rate, $\upsilon=0.5$mL/min, then $\mathrm{Pe}=0.75$. The micro-channel wall is submitted to forced convection with air ($h=50$ W/m^2°C), thus $Bi=0.75$ in eq. (2d). Figure 2a brings the evolution of the fluid bulk temperature along the micro-channel length from 0 to $Z\ \mathrm{Pe}=200$, which corresponds to the dimensional value $z=8$cm, in three situations: (i) Neglecting the axial conduction term in both the fluid and the wall (green curve), (ii) considering the axial conduction term in both the fluid and the wall, but neglecting the effect of the adiabatic region upstream of the heat exchange section (black curve) and (iii) considering axial conduction and the participation of the upstream region (blue curve). The result confirms that in this application with a micro-channel etched on a nanocomposite substrate and low volumetric flow rate, neglecting the participation of the channel walls, axial conduction and the participation of the upstream region may lead to significant deviations on the fluid bulk temperatures. It is noticeable that for the case without axial diffusion (green curve), the transversal heat loss at each cross-section is maximized, yielding the lower values of the mean fluid temperature along the channel. When only forward axial heat diffusion is accounted for (black curve), the pre-heating effect is noticeable on the higher values of the fluid temperature at downstream axial positions. When backwards axial diffusion is considered at the upstream region (blue curve), the fluid temperatures are in between these two cases. Figure 2b shows the local Nusselt numbers, where marked differences among the estimates are observed with the three formulations. We also observe that the thermal development, with the establishment of a fully developed asymptotic Nusselt number, occurs within a very short length, but noticeable delays are observed when the axial conduction term and the conduction to the upstream region are considered, in comparison with the case without axial diffusion (green).



Figures 2 – Evolution of (a) fluid bulk temperature and (b) local Nusselt number, along the channel length, for three situations: (i) neglecting the axial conduction terms (green), (ii) considering the axial conduction terms, but neglecting the effect of the adiabatic upstream region (black), and (iii) considering the participation of the upstream region (blue).

References

[1] Cotta, R.M., Integral Transforms in Computational Heat and Fluid Flow, CRC Press, Boca Raton, FL, 1993.

[2] Knupp, D.C., Naveira-Cotta, C.P. and Cotta, R.M., Conjugated heat transfer in micro-channels with a single domain formulation and integral transforms, Proceedings of the 3rd Micro/Nanoscale Heat & Mass Transfer International Conference, Atlanta, USA, March, 2012.

[3] Knupp, D.C., Naveira-Cotta, C.P., Ayres, J.V.C, Cotta, R.M. and Orlande, H.R.B., Theoretical-experimental analysis of heat transfer in nonhomogeneous solids via improved lumped formulation, integral transforms and infrared thermography, Int. J. Thermal Sciences, in press.