ANALYSIS OF THE BOLGIANO LENGTH SCALE THE BULK OF TURBULENT RAYLEIGH-BÉNARD CONVECTION

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Summary The $\mathcal{R}a$-scaling of the Bolgiano length scale and the structure-function scaling of turbulent Rayleigh-Bénard convection in a cube are investigated by means of direct numerical simulations for $5 \times 10^6 \leq \mathcal{R}a \leq 3 \times 10^9$ and $\mathcal{P}r = 4.38$. It is observed that the local Bolgiano length scale in the bulk region does not follow a simple power-law over the range of $\mathcal{R}a$ considered, which might be due to the transition from the so-called II_u to the IV_u regime. Furthermore it is found that confining the flow in one direction can be used as a means to decrease the Bolgiano length scale in the bulk, and hence allows for studying the impact of the Bolgiano scale on the structure function scaling in thermal turbulence.

Introduction

Thermally driven flows play an important role in many natural phenomena and technical applications. Rayleigh-Bénard convection (RBC) is a well-posed idealised system that allows for studying such flows. Despite the fact that this model system has been studied extensively [1, 2], fundamental behaviours, such as whether or not the energy cascade exhibit the of BO59 scaling are still debated, see e.g. [3, 4, 5]

In homogeneous isotropic turbulence or when the temperature is a passive scalar in the flow, the scalings of the velocity and temperature structure functions are expected to follow the Kolmogorov (K41) scaling (plus intermittency corrections). However, in thermally driven flows, which play an important role in many natural phenomena and technical applications, the temperature cannot generally considered to be passive. In such a case it is expected that the Bolgiano-Obukhov (BO59) scalings apply. Hence the inertial subrange is divided into two regions for $\eta_k \ll r \ll L_B$ and $L_B \ll r \ll L$, for which K41 and BO59 scalings apply and L_B is the Bolgiano length scale.

RBC is primarily determined by the Rayleigh number $\mathcal{R}a = \alpha g H^3 \Delta T/(\nu\kappa)$ and Prandtl number $\mathcal{P}r = \nu/\kappa$, where α, ν and κ are the thermal expansion coefficient, kinematic viscosity and thermal diffusivity, respectively, and g is the gravitational acceleration. The height of the fluid layer is given by H and the temperature difference between the horizontal top and bottom plates is ΔT

Here we present the results obtained from direct numerical simulations of RBC in a cube for $\mathcal{P}r = 4.38$ and $5 \times 10^6 \leq \mathcal{R}a \leq 3 \times 10^9$. The temperature boundary conditions are adiabatic in all horizontal directions and isothermal on the top and bottom walls, respectively. No-slip and impermeability conditions are applied to all walls. The solver is described in [6, 7]. We note that the resolution of the simulations exceeds the requirements posed in [8], so that all relevant turbulent scales are resolved. This allows us to determine the scaling behaviour of the Bolgiano length scale over a wide range of $\mathcal{R}a$.

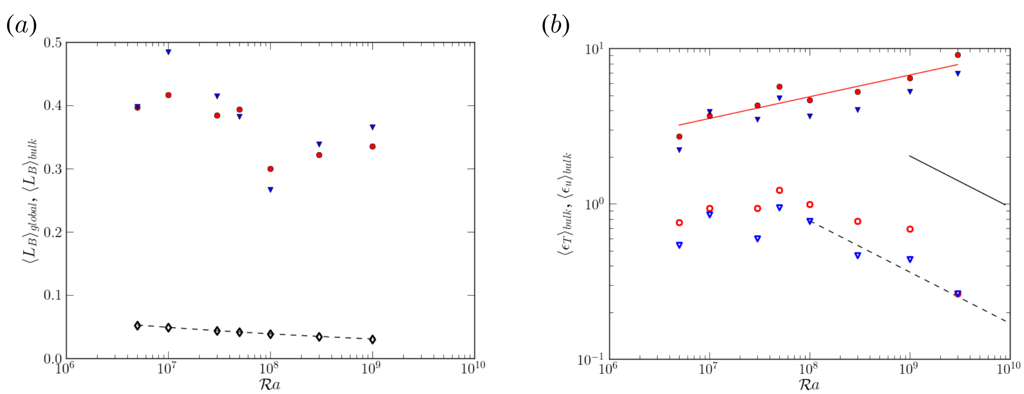

Figure 1. (a) $\mathcal{R}a$-scaling of the Bolgiano length scale in the bulk region for $\mathcal{P}r = 4.38$: $\langle L_B\rangle_{bulk,h=1/2}$ (●), $\langle L_B\rangle_{bulk,h=1/4}$ (▼) and $L_B = (\mathcal{N}u - 1)^{5/4}\mathcal{N}u^{-3/4}\mathcal{R}a^{-1/4}\mathcal{P}r^{-1/4}$ (◇); the dashed line is the power-law fit $L_B \sim \mathcal{R}a^{-0.10}$. ($b$) $\mathcal{R}a$-scaling of the thermal (open symbols) and viscous dissipation rates (filled symbols) for the same data; symbol shapes as in (a). The solid black line represents the least-squares fit for ϵ_T obtained by [9] and the dashed black line is the same scaling adjusted to the DNS data.

$\mathcal{R}a$-scaling of the Bolgiano length scale

Figure 1 (a) shows the $\mathcal{R}a$-scaling of the Bolgiano length scale L_B averaged for the bulk region $H/4 \leq x, y, z \leq 3H/4$, denoted as $\langle L_B\rangle_{bulk,h=1/2}$ and averaged over $3H/8 \leq x, y, z \leq 5H/8$, denoted as $\langle L_B\rangle_{bulk,h=1/4}$. We observe that the Bolgiano length scale in the bulk does not follow a simple power-law like the globally averaged Bolgiano length scale $\langle L_B\rangle_{global} = 0.25Ra^{-0.10}$.

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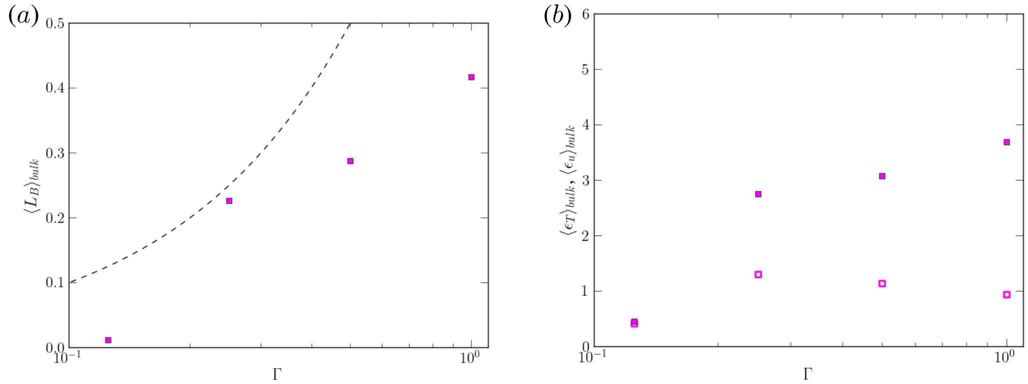

Figure 2. (a) Γ-scaling of the Bolgiano length scale in the bulk region for $\mathcal{R}a = 1 \times 10^7$ (■). (b) Γ-scaling of the viscous (filled symbols) and thermal (open symbols) dissipation rates.

The source of the complex scaling behaviour of the Bolgiano length scale lies in the $\mathcal{R}a$-dependency of the thermal dissipation rate ϵ_T presented in 1 (b), whereas ϵ_u is well approximated by a single power-law fit $\langle \epsilon_u \rangle_{bulk} \sim \mathcal{R}a^{0.14}$. It is noted that $\langle \epsilon_T \rangle_{bulk}$ first increases until $\mathcal{R}a \approx 10^8$, and then begins to decrease, approaching the scaling of the local Bolgiano scale observed by [9] for $\mathcal{R}a \geq 10^9$.
We interpret this result such that the behaviour of the thermal dissipation rate in the bulk reflects the transition from the so-called II_u to the IV_u regime [10] for $\mathcal{R}a \approx 10^8$, and hence from a boundary layer to a bulk dominated temperature field. In line with the theoretical prediction the velocity field does not exhibit such a transition.

Geometrical confinement

In order to study the influence of geometrical confinement on the Bolgiano length scale, we reduce the aspect ratio Γ of the cell in one horizontal direction, so that we obtain quasi-2D flow as Γ tends to zero. In figure 2 we present the aspect ratio dependency of the Bolgiano length scale in the bulk $\langle L_B \rangle_{bulk}$, from which can be seen that the Bolgiano length scale is decreasing once the width of the container is smaller than $\langle L_B \rangle_{bulk,cube}$ as was found for Rayleigh-Taylor convection [11].

Conclusions

In the present study we have shown that the $\mathcal{R}a$-scaling of the Bolgiano length scale in turbulent Rayleigh-Bénard convection exhibits a transition around $\mathcal{R}a \approx 10^8$, which we reckon is a result of the transition from the so-called II_u to the IV_u regime that is due to a change of the temperature field resulting in a different scaling of the thermal dissipation rates. As already noted by [11] the possibility to geometrically confine of the Bolgiano length scale, will be of great importance to study the impact of the Bolgiano scale on the structure function scaling in thermal turbulence, and hence allows for studying the impact of the Bolgiano scale on the structure function scaling in thermal turbulence.

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