ON THE SIZE OF CONVECTION CELLS IN AN INTERNALLY HEATED FLUID LAYER AT LOW RAYLEIGH NUMBERS

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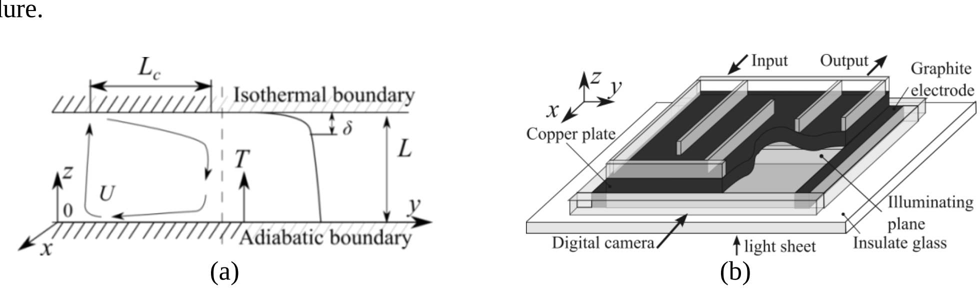
Summary The fluid dynamic scaling on the spatial wavenumber in internally heated fluid layers at low Rayleigh numbers is studied by a model based on the thermal boundary layer separation. To construct the model, experiments were conducted by measuring the temperature and velocity field with thermo-chromic liquid crystals. The results from the model explain the Rayleigh number effect on the spatial wavenumber in internally heated convection.

INTRODUCTION

Natural convection induced by internal heating governs large scale dynamics in natural systems such as the atmosphere and Earth's mantle. In industrial applications material science and nuclear reactor science also includes internally heated convections. Therefore natural convection induced by internal heating in a horizontal fluid layer has been investigated experimentally thoroughly, generally at high Rayleigh numbers (Ra_I). According to the non-linear stability analysis, the critical spatial wave-number k_c and critical Ra_I for onset $Ra_{I,c}$ is k_c = 2.63 and $Ra_{I,c}$ =1386 [1]. After the onset of convection, the fluid layer forms spatially periodical hexagon shaped convection cells. When the Ra_I increases, the hexagonal convection cells expand largely at low Ra_I regions ($Ra_{I,c} \leqq Ra_I \leqq 10\ Ra_{I,c}$). From early experiments [2], this large expansion has been in question because this is not predicted in the non-linear stability analysis and therefore is concluded as effects of experimental apparatus. However a recent experiment with careful care on these experimental apparatus effects has reported a quantitative result on the spatial wave-number decrease as the Ra_I increases [3]. In this paper, we present a 2D model based on the separation of the thermal boundary layer from the top cooling wall, in a fluid layer with internally heating, top isothermal and bottom adiabatic boundaries. This model is aimed to explain the relation between the k_c and Ra_I.

EXPERIMENTAL SETUP AND METHOD

The main parameters in this phenomenon are the Rayleigh number $Ra_I = g\beta HL^5/2\lambda\kappa\nu$ and Prandtl number $Pr = \kappa/\nu$, where H is the rate of internal heat generation per unit volume, g is gravitational acceleration, and $\beta, \lambda, \kappa, \nu$ are respectively thermal expansion coefficient, thermal conductivity, thermal diffusivity, kinematic viscosity of the working fluid. The Pr is fixed at 6.9 and Ra_I is changed by controlling H. Figure 1 shows (a) definition sketch of the experimental system and (b) the schema of the experimental apparatus. A 210 mm squared and 7 mm thick fluid layer of 0.5 wt % potassium chloride solution was internally heated by Joule heating. Joule heating was done by conducting alternating electric current from two graphite bars on parallel sides of the fluid layer. The bottom boundary of the fluid layer is a 6 mm thick glass assembly with a 0.2 mm thick gap of vacuum to improve thermal insulation and permit illumination and visualization through the bottom. The top boundary of the fluid layer is a 15 mm thick copper plate with circulating water from a thermo-static bath to regulate the temperature. The surrounding of the fluid layer and graphite electrodes are 10 mm thick Plexiglas.
The temperature was visualized with encapsulated thermo-chromic liquid crystals (TLCs). TLCs are chiral nematic liquid crystals which scatter light with wavelengths according to its temperature. The color play changes from red to blue when the temperature increases inside a designated range. The velocity field can also be obtained by following the aggregated TLCs. An adequate amount of TLCs (Japan Capsular Products, KWN-2025) were seeded into the test fluid and the vertical plane of the fluid layer was illuminated by a white light sheet (less than 2 mm thick). The illuminated vertical plane was recorded by a digital camera (Nikon, D700). The temperature was obtained from the colors by a calibration procedure.



(a) (b)

Figure 1 (a) Coordinates and main physical quantity, dashed line represents the center of convection cell. The left and right halves are velocity and temperature field, respectively. (b) Schema of experimental apparatus.

2D MODEL BASED ON THERMAL BOUNDARY LAYER SEPARATION

The model we propose is based on the distance L_c between where the thermal boundary layer starts to develop and where it separates along the top cooling boundary, which corresponds to the spatial wave-length of the convection cell. This thickness of thermal boundary layer $\delta \sim \sqrt{\kappa t}$, increases with time t when the fluid is cooled by the top cooling boundary. Since the characteristic time scale of convection is described as $t_c = L_c/U$, where the characteristic velocity of the convection cell is U, we can further express this relation to describe the critical thickness of the thermal boundary layer at its separation as follows $\delta_c \sim \sqrt{\kappa L_c / U}$. In order to predict $L_c \sim \delta_c^{\,2} U / \kappa$ with respect to Ra_I, we experimentally determined how δ_c and U depend on Ra_I.

The velocity of the convection cell U has been characterized as the magnitude of the descending flow velocity w. Previous results have reported the following relation $wL/\kappa = 6.81(Ra_I/Ra_{I,c})^{0.51}$[4] using all the data ($4Ra_{I,c} \leqq Ra_I \leqq 24Ra_{I,c}$), as shown as the dash-dot line in Fig.2 (a). However, the dependence of Ra_I on the spatial wavenumber shows weakly when this velocity relation is applied to our scaling model, which is inconsistent with experimental results. Since the convective pattern changes and the w-Ra_I slope changes before and after $Ra_I = 10Ra_{I,c}$, the approximation line we have obtained is derived from the data until $Ra_I = 10Ra_{I,c}$. This yields the following relation $wL/\kappa = 3.61(Ra_I/Ra_{I,c})^{0.90}$, as shown as the solid line in Fig.2 (a). Furthermore, δ_c is characterized by defining the critical local Rayleigh number $Ra_{\delta,c} = g\beta\Delta T\delta_c^3/\kappa\nu$, where ΔT is the temperature difference between the maximum temperature and top boundary. Here the $Ra_{\delta,c}$ is the critical local Rayleigh number for the onset of separation for the thermal boundary layer from the top boundary. Therefore by obtaining the $Ra_{\delta,c}$ and ΔT relation with respect to Ra_I, the relation between δ_c and Ra_I can be obtained. If we assume $Ra_{\delta,c}$ is constant at low Ra_I range ($Ra_{I,c} \leqq Ra_I \leqq 10\, Ra_{I,c}$), and $\Delta T \sim (Ra_I/Ra_{I,c})^{0.77}$ which is derived from power law relation of Nusselt number results[5], the following relation can be obtained. $L_c \sim \delta_c^2 U/\kappa = Ra_{\delta,c}^{\;2/3}(Ra_I/Ra_{I,c})^{0.39}$, therefore we can derive a non-dimensional wave-number and Ra_I relation as follows $kL = 2.63(Ra_I/Ra_{I,c})^{-0.39}$, as shown as the solid curve in Fig.2 (b). This relation shows a good agreement with the non-dimensional wavenumber. In this experiment, we will check the assumption of $Ra_{\delta,c}$ and ΔT by temperature field with the TLCs at the low Ra_I range ($Ra_{I,c} \leqq Ra_I \leqq 10\, Ra_{I,c}$) and report the results.

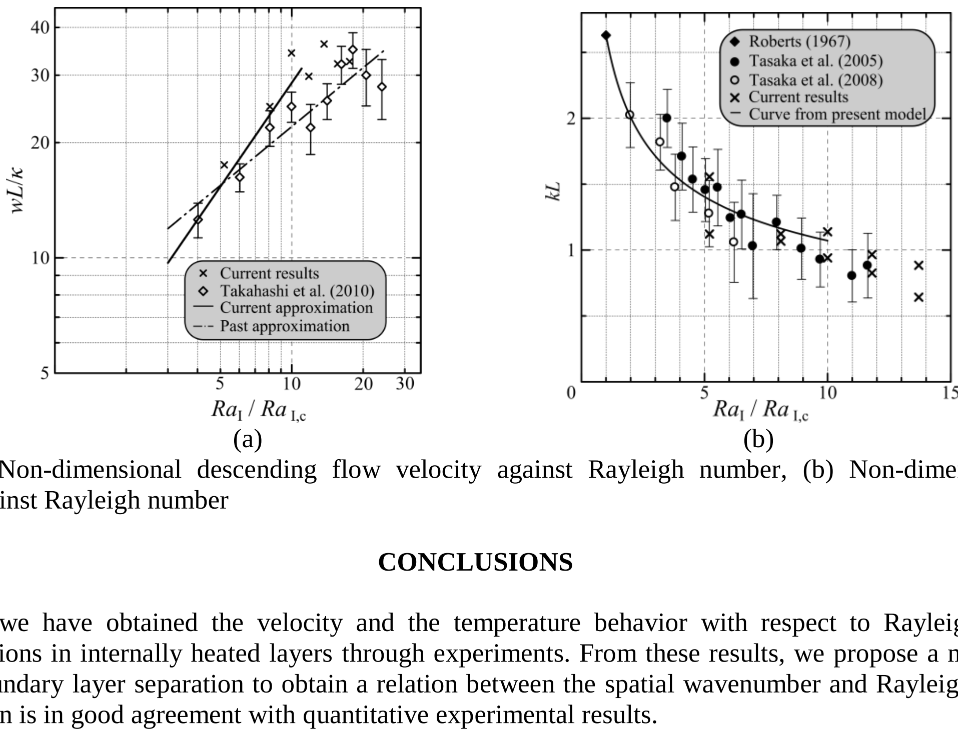


Figure 2, (a) Non-dimensional descending flow velocity against Rayleigh number, (b) Non-dimensional spatial wavelength against Rayleigh number

CONCLUSIONS

In this paper, we have obtained the velocity and the temperature behavior with respect to Rayleigh number for convective motions in internally heated layers through experiments. From these results, we propose a model based on the thermal boundary layer separation to obtain a relation between the spatial wavenumber and Rayleigh number. The obtained relation is in good agreement with quantitative experimental results.

References

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