THERMAL BOUNDARY LAYER INSTABILITIES IN VISCOUS FLUIDS

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Summary We study the development and local characteristics of thermal boundary layer (TBL) instabilities in newtonian fluids whose viscosity depends strongly on temperature, for Rayleigh numbers up to 10^8, Prandtl numbers up to 10^6, and viscosity ratios across the tank up to 4000. The temperature-dependence of viscosity introduces a strong asymmetry in the TBLs. At high Rayleigh numbers, hot instabilities develop as mushroom-shaped plumes while cold instabilities develop as thin fingers . Howard's phenomenological model (1964) describes well the hot instabilities formation: the TBL first grows by conduction until the local Rayleigh number calculated across it reaches a critical value Ra_c. Then a plume develops and drain the TBL. If it is completely drained, the plume detaches. Otherwise, plumes merge with their neighbours. Cold more viscous instabilites only disappear by merging. Plume lateral motions and merging occur along networks of ridges localized at the edges of the TBLs. Scaling laws are presented.

PLUMES IN LABORATORY EXPERIMENTS

Thermal convection in viscous fluids is a common occurrence both in industry (e.g. chemical reactors) and nature (e.g. atmosphere, oceans, lava lakes, planetary mantles). Besides, Rayleigh-Benard convection in a shallow cavity heated from below is a classic example of a non-linear system exhibiting a sequence of transitions towards chaos as a control parameter increases (see [1,2] for recent reviews). Geological fluids encountered in natural systems such as lava lakes or planetary mantles have a viscosity which depends strongly on temperature. Convection therefore is characterized by three dimensionless numbers [4]: the Rayleigh number, ratio of the driving thermal buoyancy forces to the resisting effects of thermal diffusion and viscous dissipation $Ra = \alpha g \Delta T H^3/\kappa\nu$; the Prandtl number, which compares the heat and vorticity diffusivities $Pr = \nu/\kappa$; the viscosity ratio across the fluid layer $\nu(T_{hot})/\nu(T_{cold})$. Here, H is the total fluid depth, ΔT is the temperature difference applied across it, g is the gravitational acceleration, $alpha$ is the thermal expansivity, κ is the thermal diffusivity and ν is the kinematic viscosity of the fluid at its bottom hot boundary temperature T_{hot}. For high Ra, plumes are developing from the TBLs and constitute the building blocks of turbulent convection [4,5,6]. However, due to their transient character, and technical limitations, the detailed sequence of local plume formation and disappearance has not yet been studied. We study here the development and local characteristics of thermal boundary layer (TBL) instabilities in sugar syrups, i.e. newtonian fluids whose viscosity depends strongly on temperature, for Rayleigh numbers $10^4 \le Ra \le 10^8$, Prandtl numbers $10^3 \le Pr \le 10^6$, and viscosity ratios across the tank $3 \le \gamma \le 4000$.

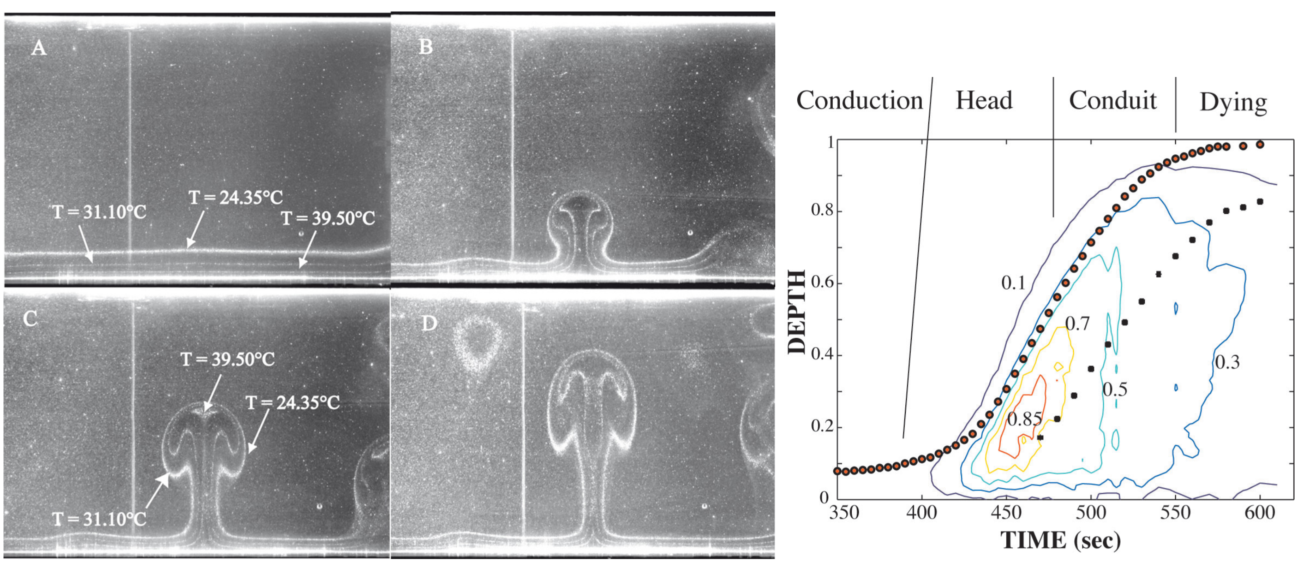

Figure 1. Evolution of the growth of hot instabilities in a layer of sugar syrup. The fluid, initially at $21.9°C$, is heated from below and cooled from above. $Ra = 8.0\ 10^5$. The horizontal bright lines at the bottom of the tank are 3 different isotherms. The vertical line is a thermocouple probe. A) $t = 560s$. Construction of the TBL where heat is transported by conduction. B) $t = 730s$. A mushroom-shaped hot plume rises from the TBL. C) $t = 790s$. The plume rises, taking hot material from the TBL which shrinks. The conduit width is about one-third of the head one. D) $t = 830s$. The plume continues to rise and cools. E) Vertical velocity along the plume axis as a function of depth and time for $Ra = 6.0\ 10^6$. The numbers give the isocontours as fraction of the maximum vertical velocity. The dots show the top, and the squares the bottom of the plume head given by the isotherm $24.35°C$.

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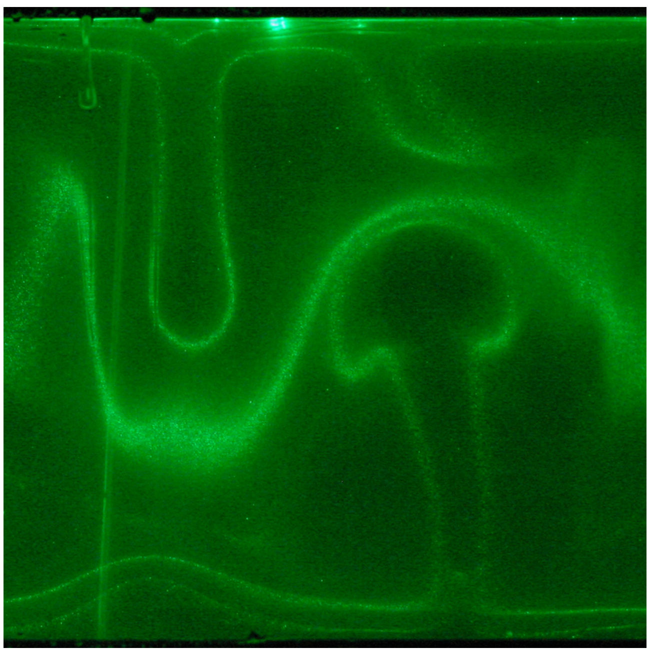
PLUMES AND RIDGES

We use thermochromic liquid crystals (TLC) slurries in combination with PIV to obtain simultaneously the temperature and the velocity fields (fig.1). The sugar layer, initially at uniform temperature Tm, is suddenly heated from below at a constant temperature T_{hot}. Several steps can be identified. At first, the isotherms (bright lines) remain horizontal and the temperature front moves away from the boundary by conduction. Then the TBL becomes unstable and generates plumes. The tip of their heads ascends initially with a constant velocity (fig.1E), and slows down when it nears the upper boundary. The point of maximum vertical velocity is initially within the head, and recedes into the stem as the head cools down . In this latter stage, the head is fed by the stem as in a plume from a fixed source. Part or all of the TBL is drained into the plume. Meanwhile, the TBL starts to locally build up again by conduction and a new cycle begins. This sequence of events totally confirm Howard's [3] phenomenological model.

The temperature-dependence of viscosity introduces a strong asymmetry in the TBLs (fig.2left). Cold instabilities are finger-shaped, and disappear only by merging. Close up on the edges of the TBL show the existence of transient and moving ridges (fig.2right). These have already reported in turbulent convection [4,5]. Plumes traveling and merging occur only on those ridges. We show that these are inherent features of laminar TBL.



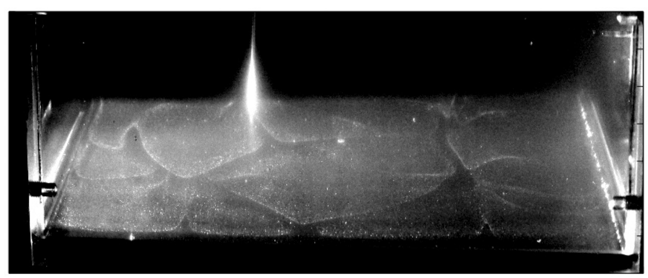


Figure 2. Left: Hot and cold TBL instabilities for sugar syrup cooled from above and heated from below ($Ra = 4.7x10^6$). The viscosity contrast between the coldest (5.0°C) and the hottest fluid (51.9°C) is 116. Right: Horizontal cross section of the same tank just at the upper edge of the bottom hot TBL. Ridges are clearly seen, on which thermal plumes travel. They will cross the whole tank only when at the junction of those ridges.

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