EFFECT OF POLYMER ADDITIVES ON HEAT TRANSPORT IN BOUNDARY LAYER FLOW

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Summary We study the effect of polymer additives on heat transport in steady-state boundary layer flow above a heated horizontal plate. Our results demonstrate that the action of the polymers, via their stretching, is equivalent to producing a space-dependent effective viscosity, which increases from the zero-shear value at the the plate. We further show that such an effective viscosity leads to a decrease in the horizontal velocity near the plate, which in turn leads to a reduction in the heat transport. Our results may thus explain a recent experimental observation of a reduction in heat transport by polymer additives in turbulent Rayleigh-Bénard convection.

INTRODUCTION

A recent experimental study reported [1] that adding polymers to turbulent Rayleigh-Bénard (RB) convection of water at moderate thermal forcing (Rayleigh number about 10^{10}) reduces the heat transport. In turbulent RB convection, there is a balance [2] between heat transport and the energy and thermal dissipation rates. For moderate thermal forcing, contributions to the dissipation rates from the boundary layers are significant [3] suggesting that the observed reduction in heat transport is due to the interaction of polymers with the boundary layer flow. The classical Prandtl-Blasius theory for steady-state boundary layer flow above a heated plate has been applied to the fluctuating boundary layers in turbulent RB convection with considerable success [4]. As a first attempt to understand the experimental observation, we study the effect of polymer additives on heat transport in steady-state Prandtl-Blasius boundary layer flow above a heated plate.

THE PROBLEM

For steady-state Prandtl-Blasius boundary layer flow above a heated plate, the equations of motion are:

$$v_x\partial_x v_x + v_y\partial_y v_x = \nu\partial^2_{yy}v_x\ ; \qquad v_x\partial_x T + v_y\partial_y T = \nu\partial^2_{yy}T \tag{1}$$

where $v_x(x,y)\hat{x} + v_y(x,y)\hat{y}$ and $T(x,y)$ are the velocity and temperature fields, $\hat{x}$ and $\hat{y}$ are respectively the directions along and away from the plate, and ν is the kinematic viscosity of the fluid. v_x and v_y vanish at the plate and v_x merges with the uniform mainstream velocity U far away from the plate. The plate is at a temperature T_1 higher than the ambient temperature T_0 far away from the plate. In the presence of the polymers, there is an additional term $\partial_y\mathcal{T}_{xy}$ in the velocity equation. Using the Oldroyd-B model [5], the polymer stress $\mathcal{T}_{ij} = (\nu_p/\tau)(R_{ij} - \delta_{ij})$. Here $R_{ij}(x,y) \equiv \langle d_i d_j/\rho_0^2\rangle$, averaging over polymers in a small region around (x,y), $\vec{d}$, ρ_0, and τ are respectively the end-to-end distance, equilibrium length, and relaxation time of the polymers, and ν_p is the polymer contribution to the zero-shear viscosity of the solution. Following [6], we take $\tau = \tau_0(1+aR)/(1+a)$ to model the increase of τ with polymer stretching, where $R \equiv (R_{xx}+R_{yy})^{1/2}$, τ_0 is the relaxation time in the unstretched state, and $a > 0$ is a parameter.

Introduce $\xi \equiv y\sqrt{U/(\nu_0 x)}$, where $\nu_0 = \nu + \nu_p$ is the zero-shear viscosity of the polymer solution, the stream function $\Psi(x,y) \equiv \sqrt{\nu x U}\phi(\xi)$ with $v_x = \partial_y\Psi$ and $v_y = -\partial_x\Psi$, and write $T(x,y) = T_0 + (T_1 - T_0)\theta(\xi)$. Taking x approximately equal to the length L of the plate in v_y, ∂_x and ∂_y, the transformation of ξ leads to a similarity solution given by:

$$-\frac{1}{2}\phi\phi_{\xi\xi} = (1-\gamma)\phi_{\xi\xi\xi} + \frac{\gamma}{\mathrm{Wi}\sqrt{\mathrm{Re}}}\frac{d}{d\xi}\left[\frac{(1+a)R_{xy}}{1+aR}\right]\ ; \qquad 2\theta_{\xi\xi} + \phi\theta_\xi\mathrm{Pr} = 0 \tag{2}$$

where $\mathrm{Wi} \equiv \tau_0 U/L$, $\mathrm{Re} \equiv UL/\nu_0$, $\mathrm{Pr} \equiv \nu_0/\kappa$ are respectively the Weissenberg, Reynolds, and Prandtl numbers, and $\gamma \equiv \nu_p/\nu_0$ depends on the polymer concentration. We assume that polymers do not change U. Heat transport is measured by the Nusselt number (Nu) and $\mathrm{Nu} = \sqrt{\mathrm{Re}}\,[-\theta_\xi(0)]$ for Prandtl-Blasius flow. We focus on the possible effects due to the non-Newtonian nature of the polymers and compare Nu to Nu_0, the value for a Newtonian fluid with the *same* kinematic viscosity ν_0. $\mathrm{Nu}/\mathrm{Nu}_0 = \theta_\xi(0)/\theta^{(B)}_\xi(0)$, where $\theta^{(B)}$ and $\phi^{(B)}$ are the solutions to Eq. (2) with $\gamma = 0$.

The components of the dimensionless polymer end-to-end distance $l_i = d_i/\rho_0$, $i = x, y$, obey the differential equations:

$$\phi\frac{dl_x}{d\xi} = \frac{(1+a)(l_x - \cos\alpha)}{\mathrm{Wi}(1+aR)} + \xi\phi_{\xi\xi}l_x - 2\sqrt{\mathrm{Re}}\phi_{\xi\xi}l_y\ ; \qquad \phi\frac{dl_y}{d\xi} = \frac{(1+a)(l_y - \sin\alpha)}{\mathrm{Wi}(1+aR)} + \frac{\xi^2\phi_{\xi\xi}}{2\sqrt{\mathrm{Re}}}l_x - \xi\phi_{\xi\xi}l_y \tag{3}$$

The equilibrium magnitude of $\vec{l}$ is 1 for the polymers but each polymer can have a different angle of equilibrium orientation, α, which has a uniform distribution in $[0, 2\pi]$. An average over the polymers is thus equivalent to an average over α. This average can be evaluated analytically only in the limit of small Wi: $R_{xy} = \langle l_x l_y\rangle_\alpha = \mathrm{Wi}\sqrt{\mathrm{Re}}[(1+aR)/(1+a)]\phi_{\xi\xi} + \langle(l_x - \cos\alpha)(l_y - \sin\alpha)\rangle_\alpha$. For a general Wi, we expect the same form to hold and approximate the average by

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$(\hat{l}_x - l_0)(\hat{l}_y - l_0)$, its value evaluated at $\alpha = \pi/4$, with $l_0 = 1/\sqrt{2}$. Thus

$$R_{xy} \approx \mathrm{Wi}\sqrt{\mathrm{Re}}\frac{(1+aR)}{(1+a)}\phi_{\xi\xi} + (\hat{l}_x - l_0)(\hat{l}_y - l_0) \equiv \mathrm{Wi}\sqrt{\mathrm{Re}}\frac{(1+aR)}{(1+a)}\phi_{\xi\xi}(1+g) \tag{4}$$

The first term on the RHS is $\tau\partial_y v_x$ thus $\partial_y[(\nu_p/\tau)R_{xy}] \equiv \partial_y[\nu_{\mathrm{eff}}(\xi)\partial_y v_x]$, demonstrating that the action of the polymers is equivalent to producing a space-dependent viscosity $\nu_{\mathrm{eff}} = \nu_p(1+g)$. With Eq. (4), the ϕ-equation in Eq. (2) becomes

$$2\phi_{\xi\xi\xi} + \phi\phi_{\xi\xi} + 2\gamma d(g\phi_{\xi\xi})/d\xi = 0 \tag{5}$$

We solve Eqs. (3), (4) and (5) at $\alpha = \pi/4$, fixed values of Re and a by iteration to obtain $\phi(\xi)$ and $g(\xi)$ for the polymer-laden flow. With this $\phi(\xi)$, $\theta(\xi)$ is obtained, and Nu is then found from $\theta_\xi(0)$.

RESULTS AND DISCUSSIONS

We find that $g(\xi)$ increases from zero at the plate up to a certain maximum then decreases rather rapidly back to zero far away from the plate. We expect this increase in the viscosity leads to a decrease in v_x near the plate, and this is indeed found. Equation (2) implies $[-\theta_\xi(0)]^{-1} = \int_0^\infty dt\,\exp[-\frac{\mathrm{Pr}}{2}\int_0^t \phi(s)ds]$ giving Nu as a functional of ϕ. Define $\Phi(\xi) \equiv \int_0^\xi \phi(s)ds$, we can calculate the change in Nu, $\delta\mathrm{Nu} = \mathrm{Nu} - \mathrm{Nu}_0$, due to a change in Φ caused by the polymers:

$$\delta\mathrm{Nu} = \frac{\mathrm{Pr}}{2\sqrt{\mathrm{Re}}}\,\mathrm{Nu}_0^2 \int_0^\infty \exp[-\mathrm{Pr}\,\Phi(s)/2]\delta\Phi(s)ds \tag{6}$$

Now $\int_0^\xi v_x/U d\xi = \phi(\xi)$. A decrease in v_x therefore implies a decrease in ϕ and thus an $\delta\Phi < 0$, which leads to $\delta\mathrm{Nu} < 0$, i.e., $\mathrm{Nu}/\mathrm{Nu}_0 < 1$ using Eq. (6). A decrease in v_x near the plate also indicates a reduction in the mass throughput near the plate, suggesting an enhancement in friction drag. The drag coefficient $C = 2\nu_0\partial_y v_x\big|_{y=0}/U^2 = \phi_{\xi\xi}(0)/(2\sqrt{\mathrm{Re}})$, and we are interested in the ratio $C/C_0 = \phi_{\xi\xi}(0)/\phi_{\xi\xi}^{(B)}(0)$. It is indeed found that $C/C_0 > 1$. A reduction in Nu and an enhancement in C is indeed observed, as shown in Fig. 1.

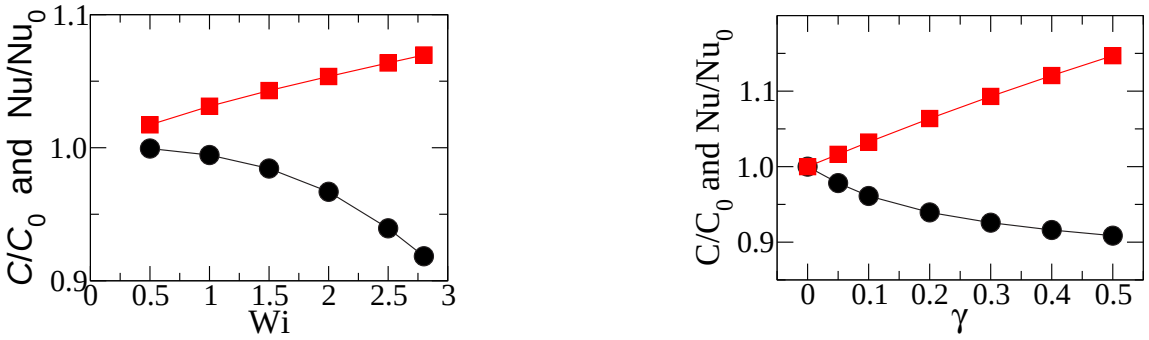

Figure 1. C/C_0 (squares) and $\mathrm{Nu}/\mathrm{Nu}_0$ (circles) as a function of Wi (at $\gamma = 0.2$) and γ (at Wi = 2.5). Pr=4.4, Re=4900, and a=0.01.

CONCLUSIONS

We have studied heat transport in steady-state boundary layer flow with polymers in forced convection above a heated plate. Our results demonstrate that the physical effect of the polymers is equivalent to producing a space-dependent effective viscosity, which increases near the plate. Because of this increase in the effective viscosity, the horizontal velocity near the plate is reduced leading to a reduction in heat transport. Our results may thus explain the recent experimental observation of a reduction in heat transport by polymer additives in turbulent RB convection.

Acknowledgements

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