TURBULENT CONVECTION AT HIGH RAYLEIGH AND TAYLOR NUMBERS

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Summary
Measurements of turbulent heat transport at very high Rayleigh and Taylor numbers are reported, with both steady and time-varying rotation. The highest Rayleigh (Taylor) number attained for steady rotation was 4.3×10^{15} (3×10^{15}). The Rossby number varied from 0.4 to 1.6. For these largest values of Ra and Ta the Prandtl number Pr was 5.9, similar to that of water; however, in contrast to the strong enhancement of heat transport for similar Pr and Ro previously reported at lower Ra and Ta, a slight suppression of the heat transport is observed here. With a periodic, time-varying rotation rate we find either suppressed or enhanced heat transport–with a sharp transition between them–depending on the value of the dimensionless amplitude of the oscillation. The transition also roughly coincides with an order unity and increasing ratio of the Ekman to thermal boundary layer thicknesses.

INTRODUCTION

A model system for many buoyancy-driven flows in nature and engineering is Rayleigh-Bénard convection (RBC) where a fluid confined in a horizontal, thin (*i.e.*, incompressible) layer, is heated from below and cooled from above. The vertical temperature difference, causing buoyancy force, is measured in terms of the Rayleigh number $Ra \equiv \alpha \Delta T g H^3/\nu\kappa$, where α is the isobaric thermal expansion coefficient of the fluid in the container, ΔT the temperature difference between the bottom and top walls, g the acceleration due to gravity and H the vertical dimension of the convection cell. The kinematic viscosity and the thermal diffusivity of the fluid are denoted, respectively, by ν and κ. The ratio ν/κ is the fluid Prandtl number, Pr. The macroscopic flow becomes turbulent for large Ra.

One particular feature of large scale geophysical and astrophysical flows is that convection is always accompanied by rotation, which introduces the Coriolis force. The relative strength of the dimensioned rotation, Ω_D (in rad s^{-1}) is given by its non-dimensional form, $\Omega = (2\Omega_D H^2/\nu)$, which is a measure of the ratio of Coriolis to viscous forces and is the inverse of the Ekman number, Ek. It is customary to also consider the Taylor number $Ta = \Omega^2$. A measure of the relative importance of Coriolis and buoyancy forces is given by the convective Rossby number $Ro = [Ra/(PrTa)]^{1/2}$ which can also be interpreted as the ratio of the rotation period to the buoyancy time scale.

It was observed by Rossby[1], and in more recent studies[2-9], that rotation enhances heat transport for moderate Pr. This enhancement was attributed to the extra vertical circulation resulting from Ekman suction at the boundaries, although Kunnen *et al.* [5] argued that large rotation rates would weaken and even reverse this tendency. Furthermore, Zhong *et al.*[9] observed that the enhancement of heat transport diminishes with increasing Ra and decreasing Pr. The present results at considerably higher Ra and Ta, and at moderately large Pr, show that the change in heat transfer under steady rotation in fact becomes vanishingly small but is of the opposite sign; i.e. there is a slight suppression.

APPARATUS

The apparatus is essentially the same as that described in earlier works[10]. Briefly, a thin-wall (0.267 cm) stainless steel cylinder of diameter 50 cm confines the fluid laterally while two 3.8 cm thick plates made of oxygen-free high-conductivity annealed copper, having a thermal conductivity of the order 1 kW $\mathrm{m}^{-1}\mathrm{K}^{-1}$ at the measurement temperature, provide top and bottom thermal boundaries with a separation distance of 100 cm between them. The diameter to height aspect ratio, Γ, is thus 1/2. Rotation was provided by placing the entire apparatus on a thick steel plate attached to a large slewing ring. Platform speed was measured and held constant by means of an optical encoder on the motor shaft and associated PID control. All measuring and data storage instruments are located on the rotating platform with the main power transferred via a heavy-duty rotating contact slip ring. For these data relatively modest rotation rates up to 1.05 rad s^{-1} were used, to avoid vibrations and large centrifugal forces; large magnitudes of Ta were obtained mainly by working under conditions of small kinematic viscosity.

RESULTS

Steady rotation

At the highest Ra and Ta studied, $Pr = 5.9$ similar to that of water. Specifically, for Ra (Ta) = 4.3×10^{15} (3×10^{15}), $Pr = 5.9$, and over a range 0.4 < Ro < 1.6 the Nusselt number (Nu) variation with Ta and Ro is summarized by the following relations:

$$Nu = 0.982 Nu_0 Ro^{0.024\pm0.001}, \quad (1)$$

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$$Nu = 1.48 Nu_0 Ta^{-0.012 \pm 0.001}, \qquad (2)$$

where Nu_0 is the value of Nu for the same Ra without rotation. These relations indicate a weak *suppression* of heat transport with *increasing* rotation, in contrast with the results for the same Pr and Ro at lower Ra and Ta (i.e. corresponding to previous experiments using water).

Time-varying rotation

The effects of a periodic modulation of the rotation rate, resulting in sequential cycles of spin-up and spin-down, were also investigated. While such processes have been examined near the convective onset[11, 12], relatively little is known in strongly turbulent convective systems where relatively small diffusive boundary layers interface with a well-mixed interior[13]. For a series of measurements the rotation rate was varied sinusoidally in time[14] as:

$$\Omega_D(t) = \Omega_D^* cos(\omega t). \qquad (3)$$

The rate at which the rotational speed is varied is set by ω, whose inverse was chosen to be on the order of the large scale circulation time in the cell—this being one plausible time scale for turbulent convection. Very large or small ω would be characterized, respectively, by dominant Stokes layer development and associated viscous waves, or quasi-steady conditions. An important parameter for the present experiments is Ω_D^*, which is the peak (dimensioned) rotation rate experienced by the flow during a cycle. A dimensionless rotation amplitude Ω^* serves as a new dynamical parameter, defined as

$$\Omega^* = [\frac{2\Omega_D^* H^2}{\nu}] \qquad (4)$$

For increasing Ω^*, a weak suppression of Nu was observed, consistent with results from steady rotation; however, above a critical value of roughly $\Omega_c^* \sim 10^7$, a sharp transition to enhanced heat transport was observed, where the effect on the heat transfer was positive and strongly increasing with increasing Ω^*. The transition is coincident with the ratio of the Ekman to thermal boundary layer thickness approaching unity, a dynamical parameter recently suggested by King, et al.[15] for characterizing steady rotating turbulent convection. Here it should be noted that the same conditions exist at high Ra in the case of fixed rotation rate without any enhancement of Nu. Finally, neither the Ekman layer thickness nor the thermal boundary layer thickness has been directly measured, nor do their estimated values take into account the time-varying emission of plumes and the existence of strong fluctuations.

CONCLUSIONS

In conclusion, a weak suppression of heat transport has been observed with steady rotation at high values of Ra and Ta but moderate Pr and Ro (of order unity) which is in contrast with previous observations at similar Pr and Ro at lower Ra and Ta. It is, however, broadly consistent with the trend noted by Zhong, et al[9]. With periodic modulation of the rotation rate a sharp transition is observed between a weak suppression of heat transport at low values of the dimensionless modulation amplitude Ω^* and an increasingly large enhancement of heat transport as Ω^* exceeds a critical value of approximately $\Omega_c^* \sim 10^7$.

References

[1] Rossby, H.. 1969 A study of Benard convection with and without rotation. *J. Fluid Mech.* **36** 309-335.
[2] Zhong, F., Ecke, R.E., & Steinberg, V. 1993 Rotating Rayleigh-Benard convection asymmetric modes and vortex states. *J. Fluid Mech.* **249**, 135-159.
[3] Julien, K., Legg, S., McWilliams, J. & Werne, J. 1996 Hard turbulence in rotating Rayleigh-Benard convection *Phys. Rev. E* **53**, R5557-R5560
[4] Liu, Y. & Ecke, R.E. 1997 Heat transport scaling in turbulent Rayleigh-Benard convection: Effects of rotation and Prandtl number *Phys. Rev. Lett.* **79**, 2257-2260.
[5] Kunnen, R.P.J., Clercx, H.J.H. & Geurts, B.J. 2006 Heat flux intensification by vortical flow localization in rotating convection *Phys. Rev. E* **74** art. no. 056306.
[6] Kunnen, R.P.J, Clercx, H.J.H. & Geurts, B.J. 2008 Breakdown of large-scale circulation in turbulent rotating convection *Euro. Phys. Lett.* **84**, art. no. 24001.
[7] Oresta, P., Stringano, G. & Verzicco R. 2007 Transitional regimes and rotation effects in Rayleigh-Benard convection in a slender cylindrical cell. *Euro. J. Mech. B/Fluids* **26** 1-14.
[8] Stevens, R.J.A.M., Zhong, J.-Q., Clercx, H.J.H., Ahlers, G., and Lohse, D. 2009 Transitions between turbulent States in Rotating Rayleigh-Benard Convection. *Phys. Rev. Lett.* **103** 024503.
[9] Zhong, J-Q, Stevens, R.J.A.M., Clercx, H. J. H., Verzicco, R., Lohse, D. & Ahlers, G. 2009 Prandtl-, Rayleigh-, and Rossby-Number Dependence of Heat Transport in Turbulent Rotating Convection. *Phys. Rev. Lett.* **102**, 044502.
[10] Niemela, J.J., Skrbek, L., Sreenivasan, K.R. & Donnelly, R.J. 2000 Turbulent convection at very high Rayleigh numbers. *Nature* **404**, 837-840.
[11] Niemela, J.J., Smith, M.R. & Donnelly, R.J. 1991 Convective instability with time-varying rotation. *Phys. Rev. A* **44**, 8406-8409.
[12] Thompson, K.L., Bajaj, K.M.S., and Ahlers, G. 2002 Traveling concentric-roll patterns in Rayleigh-Beï£¡nard convection with modulated rotation. *Phys. Rev. E* **65** 046218.
[13] Niemela, J.J. & Sreenivasan, K.R. 2008 Formation of the “Superconducting" core in turbulent thermal convection. *Phys. Rev. Lett.* 100, art. no. 184502
[14] Niemela, J.J., Babuin, S., & Sreenivasan, K.R. Turbulent rotating convection at high Rayleigh and Taylor numbers,*J. Fluid Mech.* **649** , 2010, 509 - 522.
[15] King, E.M., Stellmach, S., Noir, J., Hansen, U., & Aurnou, J.M. 2009 Boundary layer control of rotating convection systems. *Nature* **457** 301-304.