UNSTEADY NATURAL CONVECTION IN A RESERVOIR INDUCED BY RAMPED ISOTHERMAL SURFACE HEATING

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Summary The present investigation is concerned with natural convection induced by ramped isothermal heating at the water surface in a wedge-shaped coastal domain. Results of numerical simulations suggest that the flow scenario depends on the comparison between the ramp duration P and the characteristic time scales. The short and the long ramp scenarios are revealed for both the near shore conductive region and the offshore convective region.

INTRODUCTION

Field experiments in the coastal regions [1-4] indicate that natural convection significantly reduces the time to replace the water, and therefore the risk of eutrophication and pollution. The measured velocity associated with this process is of the order of 5 cm/s. The environmental significance has motivated increasing effort in understanding the flow mechanisms for various thermal forces. For constant radiation heating, theoretical quantification of the thermal flow has been achieved through zero-order asymptotic solutions and scaling analysis [5-7]. For these investigations, the thermal forcing is in the form of either a specified heat flux, or an internal heating source, or a combination of both. In reality, apart from this, the thermal forcing may also be resulted from a temperature difference between the water body and the atmosphere. For such a case, a specified temperature rather than heat flux is more appropriate. A literature survey indicates that studies on isothermal heating in a wedge domain are very scarce. The present study aims for the case of ramped isothermal surface heating at the water surface.

MODEL FORMULATION

Here, a two-dimensional reservoir consisting of one sub-region with a sloping bottom with a slope of A and the other sub-region with uniform depth is considered (figure 1). With the Boussinesq assumption, the two-dimensional Navier-Stokes and energy equations governing the flow and temperature evolution within the wedge are written in (1)-(4).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\nu\nabla^2 u, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\nu\nabla^2 v+g\beta\left(T-T_0\right), \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\kappa\nabla^2 T. \tag{4}$$

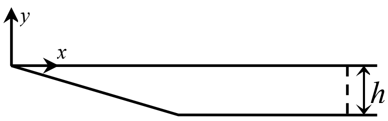

Figure 1 Sketch of the flow domain and the coordinate system.

Here u and v are the horizontal and vertical velocity components, respectively and p is the pressure. The parameters ρ_0, κ, ν and β are respectively the density, thermal diffusivity, kinematic viscosity and thermal expansion coefficient of the fluid at a reference temperature T_0. The flow is initially stationary and isothermal at a temperature of T_0. An adiabatic and rigid no-slip boundary condition is assumed for the bottom, whereas the water surface is assumed stress free ($\partial u/\partial y=0$ and v = 0). An open boundary condition is considered for the endwall ($\partial u/\partial x=0$, v = 0), assuming that any backflow is at the initial temperature of T_0. The temperature at the water surface first increases linearly with time over a certain time duration of P and then becomes constant, expressed as: $T_{y=0}=\begin{cases}T_0+(\Delta T/P)\cdot t & 0\le t\le P\\ T_0+\Delta T & t>P\end{cases}$.

NUMERICAL SIMULATIONS

The governing equations (1)-(4) along with the specified boundary and initial conditions are solved numerically using a finite-volume method. The SIMPLE scheme is adopted for pressure-velocity coupling; and the QUICK scheme is applied for spatial derivatives. A second-order implicit scheme is applied for time discretization in calculating the transient flow. To generalize the results, the results are presented in a normalized way. The respective scales for the normalization are: the length scale $x, y\sim h$; the time scale $t\sim h^2/\kappa$; the velocity scale $u, v\sim \kappa/h$; and the pressure gradient scale $p_x, p_y\sim \rho g\beta\Delta T$. The non-dimensional temperature is τ defined as $(T-T_{y=0})/\Delta T$. And thus τ represents the temperature difference between the local and the water surface temperatures.

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A comparison between the ramp duration P and other characteristic time scales pertinent to the flow is expected to reveal different scenarios of the flow development. For the conductive region near shore, the characteristic time scale is the time for the thermal boundary layer to reach the local water depth. For the convective region, the characteristic time scale is the steady state time. The possible scenarios are shown by the simulations results for the conductive and the convective region respectively.

Conductive region

Two possible scenarios of shore ramp and long ramp are plotted in figure 2. It is clear that the variation pattern of temperature and velocity with time is different for these two scenarios. For the short ramp scenario (figure 2*a* and *c*), a steady state is not reached within the ramp duration. After the ramp finishes at t = 0.04, the temperature difference and velocity gradually decreases to zero. For the long ramp scenario (figure 2*b* and *d*), a steady temperature difference and velocity is observed up until the ramp finishes at t = 0.4, For both scenarios, the flow eventually becomes isothermal and stagnant.

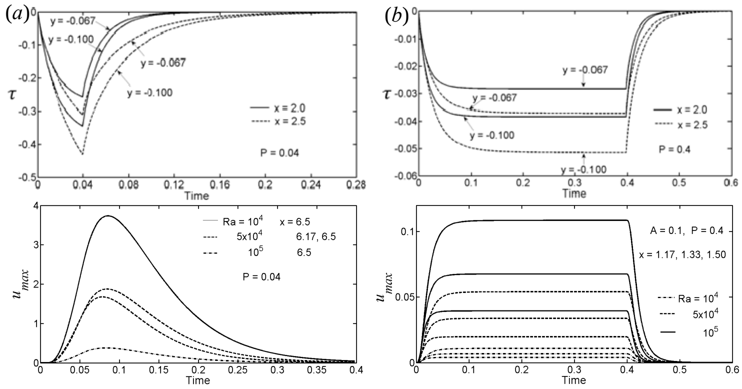

Figure 2 Time series at different positions and Ra (*a*) Temperature for the short ramp scenario P = 0.04 (*b*) Temperature for the long ramp scenario P = 0.4. (*c*)Velocity for the short ramp scenario (*d*) Velocity for the long ramp scenario

Convective region

The time series of the maximum velocity along the vertical line of x = 3.5 are plotted in figure 3 for the two possible flow scenarios along with the result of constant heating for the same Rayleigh number. Figure 11 suggests that irrespective of the ramp duration, the ramp heating leads to the same steady state velocity as the corresponding constant heating case. For the short ramp case (figure 3a), the flow velocity increases smoothly both before and after the ramp finishes and gradually reaches the steady state afterwards. For the long ramp case (figure 3b), there is a distinct change in the growth curve of the flow velocity within the ramp duration. This change is due to the fact that a quasi-steady state is reached. This variation of the velocity growth rate agrees well with the scaling prediction, as it can be derived that the growth rate decreases from $t^{5/2}$ at the initial stage to $t^{2/5}$ at the quasi-steady stage. The flow becomes steady soon after the ramp finishes.

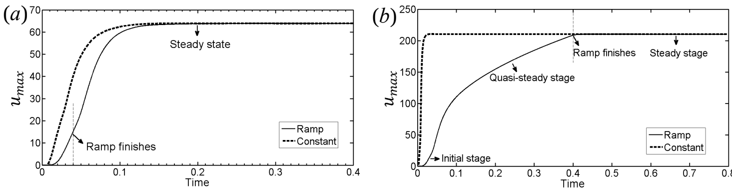

Figure 3 The time series of horizontal velocity along the line of x = 3.5 for the two different scenarios, the solid and dashed lines are the results of the ramp heating case and the corresponding constant heating case respectively (a) short ramp P = 0.04, Ra = 10^6 (b) long ramp, $Ra = 2 \times 10^7$.

CONCLUSIONS

Different flow scenarios are revealed by comparing the ramp duration with the characteristic time scales of the flow development. For the conductive region, the characteristic time scale is the time for the thermal boundary layer to reach the local depth. For the convective region, it is the steady state time when convection balances conduction. For both regions, if the ramp duration is sufficiently long, a quasi-steady state is reached within the ramp duration. For the conductive region, during the quasi-steady stage, the local temperature increases at the same rate as the water surface temperature and the flow velocity is steady. After the ramp finishes, the flow eventually becomes isothermal and stationary. For the convective region, the growth rate of velocity in the quasi-steady stage decreases significantly compared to the initial stage. The flow velocity becomes constant soon after the ramp finishes. For both scenarios, the steady state velocity for the ramp heating case is the same as the corresponding constant heating case.

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