GENERALIZED AUTOCORRELATION FUNCTION IN RAYLEIGH-BÉNARD CONVECTION

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Summary A generalized autocorrelation function is proposed to extract scaling exponent from a temperature time series obtained from Rayleigh-Bénard convection system with statistical moment $q \leq 4$. The estimated scaling exponent $\zeta_A(q)$ crosses the Kolmogorov-Obukhov-Corssin prediction when $q = 1$ and crosses the Bolgiano-Obukhov prediction when $q = 2$. The $\zeta_A(q)$ is below BO59 scaling when $q > 2$. A scale contribution analysis shows that the second-order structure function is contaminated by large-scale-circulation.

INTRODUCTION

For the Rayleigh-Bénard (RB) convection turbulent system, the flow is dominated by buoyant forces mainly via thermal plumes. According to Bolgiano and Obukhov arguments, above the so-called Bolgiano length scale ℓ_B, instead of the Kolmogorov-Obukhov-Corrsin (KOC) scaling $S_q^\theta(r) \sim r^{q/3}$ for the passive scalar, the Bolgiano-Obukhov (BO59) scaling $S_q^\theta(r) \sim r^{q/5}$ for the active scalar is expected for the temperature, in which $S_q^\theta = \langle |\Delta_r \theta(x)|^q \rangle$ is the qth order structure function (SF), $\Delta_r \theta(x) = \theta(x+r) - \theta(x)$ temperature increment, and r the separation length scale [1]. However, there exist several difficulties to identify the BO59 scaling in both temporal and spatial domains [1]. These difficulties are recognized as the anisotropy and the inhomogeneity of the flow, in which the BO59 argument may fail, insufficient high Rayleigh numbers, intermittent corrections, the presence of a shear, and the lack of clear separation between Bolgiano length scale ℓ_B and the height of the cell L, etc. [1]. Thus whether the BO59 scaling exists in turbulent RB system is still a major challenge, see more detail in Ref. [1].

GENERALIZED AUTOCORRELATION FUNCTION

We introduce here a generalized autocorrelation function (GACF) by considering a sign statistics, i.e.,

$$\Gamma_q(\tau) \equiv \langle \mathrm{sp}\,(Y_\tau(t))^{q/2} \rangle, \quad \mathrm{sp}(Y_\tau)^q \equiv |Y_\tau|^q \times \mathrm{sgn}(Y_\tau) \tag{1}$$

in which $Y_\tau(t) \equiv \Delta x_\tau(t+\tau)\Delta x_\tau(t)$, and $\mathrm{sp}(Y_\tau)^q$ is the sign power, $\mathrm{sgn}(Y_\tau)$ is sign of Y_τ. When $q = 2$, Eq.(1) naturally recovers the traditional definition of the ACF. For a scaling time series, it has been proved for $q = 2$ analytically that a power law behavior $\Gamma_2(\tau) \sim \tau^\beta$, e.g., $\beta = 2H$ in which H is the so-called Hurst number [2]. We postulate here the power law behavior

$$\Gamma_q(\tau) \sim \tau^{\zeta_A(q)} \tag{2}$$

in which $\zeta_A(q)$ is the scaling exponents. The above equation has been validated for H-mono-fractal process, which is not shown here.

EXPERIMENTAL RESULTS AND DISCUSSION

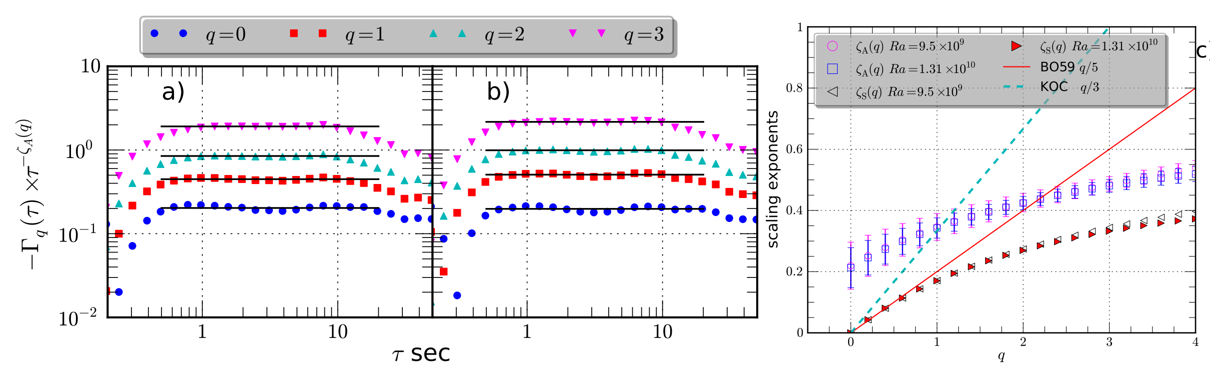

Figure 1. a) measured compensated generalized autocorrelation function $\Gamma_q(\tau)\tau^{-\zeta_A(q)}$ for $Ra = 9.5 \times 10^9$, and b) $Ra = 1.31 \times 10^{10}$ and c) the corresponding scaling exponents $\zeta_A(q)$. For comparison, scaling exponents provided by structure function, KOC theory and BO59 theory are also shown.

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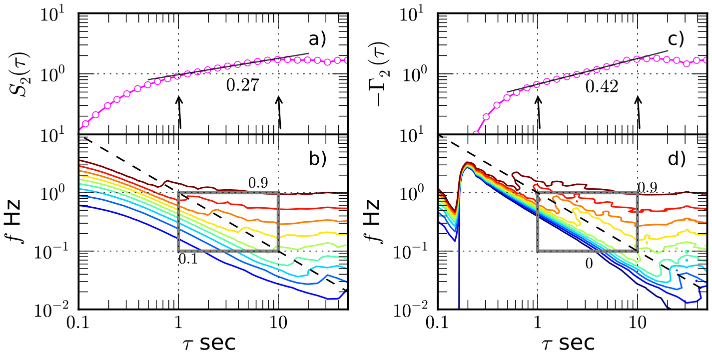

Figure 2. a) measured second-order structure function, b) relative contribution function, c) second-order autocorrelation function and d) relative contribution function. The solid line in a) and c) is a least square fitting of the power law. The power-law range in b) and d) is demonstrated by a square. Note that a scaling behavior indicates a series contour lines to be in parallel with $f = 1/\tau$ (dashed line).

The experiments were performed by Prof. Xia's group in the Chinese University of Hong Kong. The details of the experiments have been described elsewhere [3]. We thus describe some main parameters below. The inner diameter of the cylindrical cell is $D = 19.0\,\text{cm}$ and the height is $L = 19.6\,\text{cm}$, thus the aspect ratio is $\Gamma \equiv D/L \simeq 1$. Water was used as the working fluid and measurements were made at Rayleigh number $Ra \equiv \beta g \Delta L^3/\nu\kappa = 9.5 \times 10^9$ and 1.31×10^{10} with g being the gravitational acceleration, Δ the temperature difference across the fluid layer, and β, ν, and κ being, respectively, the thermal expansion coefficient, the kinematic viscosity, and the thermal diffusivity of the working fluid (water). The local temperature was measured at $8\,\text{mm}$ from the sidewall at midheight by using a small thermistor of $0.2\,\text{mm}$ diameter and $15\,\text{ms}$ time constant. Typically, each measurement of temperature lasted 20 hours or longer with a sampling frequency $64\,\text{Hz}$, ensuring that the statistical averages are adequate. For the present experiments, t_B is found to be $\sim 1\,\text{sec}$.

Figure 1 shows the compensated GACF $\Gamma_q(\tau)\tau^{\zeta_A(q)}$ for a) $Ra = 9.5 \times 10^9$ and b) $Ra = 1.31 \times 10^{10}$ with $0 \leq q \leq 4$. Clear plateau is observed in the range $1 < \tau < 10\,\text{sec}$. The corresponding scaling exponent $\zeta_A(q)$ is shown in Fig. 1 c). Note that the $\zeta_A(0) \simeq 0.21$, which might be an effect of finite Rayleigh (resp. Reynolds) number. The estimated $\zeta_A(q)$ crosses the KOC prediction when $q = 1$. The first-order KOC-like scaling has been observed previously by considering a scaling behavior of the maximum probability density function (pdf) of temperature increments [5]. Estimated $\zeta_A(q)$ crosses the BO59 prediction when $q = 2$. This BO59-like scaling can also be obtained by applying the pdf scaling approach to Y_τ (not shown here). Note that the SF scaling exponent $\zeta_S(q)$ is below the BO59 prediction when $q > 1$.

To understand why the second-order SF can not gain the BO59-like scaling, we introduce here a relative cumulative function $\mathcal{Z}(\tau, f)$

$$\mathcal{Z}(\tau, f) = \frac{\int_0^f E(f')\mathcal{W}(\tau, f')df'}{\int_0^{+\infty} E_(f')\mathcal{W}(\tau, f')df'} \tag{3}$$

in which $E(f)$ is Fourier power spectrum of temperature, $\mathcal{W}(\tau, f) = (1 - \cos(2\pi f\tau))$ and $\mathcal{W}(\tau, f) = (1 - \cos(2\pi f\tau))\cos(2\pi f\tau)$ respectively for SF and ACF [2]. It describes a relative contribution of the frequency range $0 \sim f$ [2]. Figure 2 shows a) the second-order SF $S_2(\tau)$, b) the contour plot of the measured cumulative function $\mathcal{Z}_\text{S}(\tau, f)$, c) the second-order GACF $\Gamma_2(\tau)$, and d) the measured cumulative function $\mathcal{Z}_\text{A}(\tau, f)$. The scaling range is indicated by arrows and square in both physical and frequency domains. Note that the scaling behavior indicates a series of contour lines to be in parallel with $f = 1/\tau$ (dashed line) at least in the power law behavior range. Graphically, this does not hold for the SF, which can be understood as the influence of the large-scale-circulation (LSC). For the ACF, in the power law range, the contour line is indeed in parallel with $f = 1/\tau$. The LSC effect is thus constrained. However, for the high-order statistical moments $q > 2$, the effect of LSC can not be eliminated. It means that we may not retrieve the BO59 scaling by using the classical SF since the thermal plumes and LSC oscillations are always presented themselves in the most RB cells [1, 6].

References

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