OPTIMAL TAYLOR-COUETTE: NUMERICAL RESULTS

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Summary A direct analog to Rayleigh-Bénard flow, where heat is transported from the bottom to the hot plate, is Taylor-Couette flow, where angular velocity is transported from the inner to the outer cylinder. To further elucidate this analogy, here we numerically simulate turbulent Taylor-Couette flow for independently rotating inner and outer cylinders. Reynolds numbers of $Re_i = 5 \cdot 10^3$ and $Re_o = \pm 2 \cdot 10^4$ are reached, corresponding to Taylor numbers beyond 10^7. Scaling laws for the torque and other system responses are revealed. Recent experiments with the Twente turbulent TC (T^3C) setup [1] at high Reynolds numbers have revealed an optimum transport at a certain counterrotation rate. For large enough Ta we find such an optimum at non-zero counter-rotation also in the numerics. We measure the corresponding angular velocity profiles and visualize the different flow structures for the various regimes.

INTRODUCTION

Taylor-Couette (TC) flow is the flow between two coaxial, independently rotating cylinders. As it is a closed system, global transport balances can be established. These balances give more insight into the fundamental behavior of the flow, as they reflect the interplay of bulk and boundary layer. The TC system has been shown to be in close analogy [2] to the Rayleigh-Benard (RB) system, where heat is transported from the bottom hot plate to the top cold plate. Using this analogy, the TC can be seen as a system where there is a transport of angular velocity from the inner to the outer cylinder. TC flow has been extensively investigated experimentally at both low and high Reynolds numbers [3, 4, 5, 6, 7, 8, 9, 10]. Numerical investigations however mainly concentrate in the case of pure inner cylinder rotation [11, 12, 13, 14], or counterrotation at a fixed rotation ratio [15]. Recent experiments [1] have developed on the RB analogy and shown the importance of reformulating the control parameters, resulting in a revelation of an optimum transport at a certain angular velocity ratio.

The system can be described through four parameters. These are two geometrical parameters, namely the radius ratio $\eta = r_i/r_o$ and the aspect ratio $\Gamma = L/d$, where d is the gap width and L the length of the cylinder, and two parameters for the driving of the flow inside this geometry, namely the two Reynolds numbers associated to inner cylinder $Re_i = \omega_i r_i d/\nu$ and outer cylinder $Re_o = \omega_o r_o d/\nu$. The analogy between RB and TC flow suggests [2] to express the driving parameters as Taylor number $Ta = \frac{1}{4}((1+\eta)/2)/\sqrt{\eta})^4 d^2 (r_i + r_o)^2 (\omega_i - \omega_o)^2 \nu^{-2}$ and as ratio of angular velocities $a = -\omega_o/\omega_i$. The response of the system is the r-independent angular velocity flux $J^\omega = r^3[\langle u_r \omega \rangle_{A,t} - \nu \partial_r \langle \omega \rangle_{A,t}]$ [2], where $\langle \cdot \rangle_{A,t}$ denotes an average over time and over a cylinder of radius r coaxial with both TC cylinders. Using the analogy with RB flow, the system response is best expressed in terms of a 'Nusselt' number $Nu_\omega = J^\omega / J^\omega_{lam}$.

The TC system was simulated using a finite difference DNS solver in cylindrical coordinates. The flow was taken as incompressible and cylindrical coordinates were used. A fixed geometry was taken with $\eta = 0.714$ and $\Gamma = 2\pi$. Time integration was done fractionally, using a 3rd order Runge-Kutta method in combination with the Crank-Nicholson scheme. More details of the numerical method can be found in [16]. The domain was taken to be periodic in the axial direction. Large scale parallelization was done using a combination of MPI and OpenMP.

RESULTS

The numerical results are compared to the experimental data for Nu_ω from ref. [10] for pure inner cylinder rotation $a = 0$, giving good agreement in the overlapping regime (Figure 1a). We in addition find a transition in the $Nu_\omega(Ta)$ scaling law between $Ta = 4 \cdot 10^6$ and $Ta = 1 \cdot 10^7$, which already had been reported in the numerical simulations of [12].

Figure 1b shows the dependence of $(Nu_\omega - 1)/(Nu_\omega(a = 0) - 1)$ at a fixed Ta for different a. For small Ta up to $Ta \approx 10^6$ there is a peak slightly past $a = 0$, which moves toward larger a for the highest Ta. Some anomalous points appear in the graph, because even when the simulations have the same initial conditions, different vortical states may be present. This peculiar behavior is linked to the average gradient of $\omega' = (\omega - \omega_o)/(\omega_i - \omega_o)$ in the bulk [1]. This gradient quantifies the importance of convection in the transport of angular momentum. There is a plateau of the gradient around $a = 0$. This is where the optimum transport points are located. The gradient decreases with Ta as convection becomes more important.

The importance of the neutral surface (where $\omega = 0$) is also highlighted in [1, 2]. This surface divides the flow into two regions: a Rayleigh-stable region and a Rayleigh-unstable region. Turbulent bursts push this neutral surface further away from the laminar position, increasing the unstable region. This means that the vortices can penetrate further and

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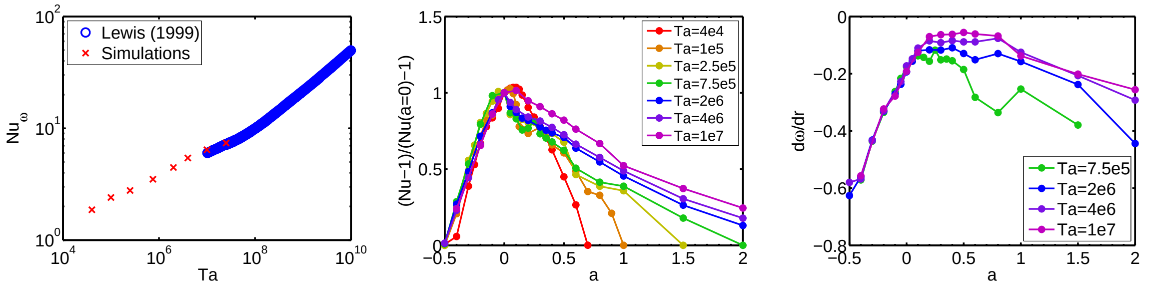
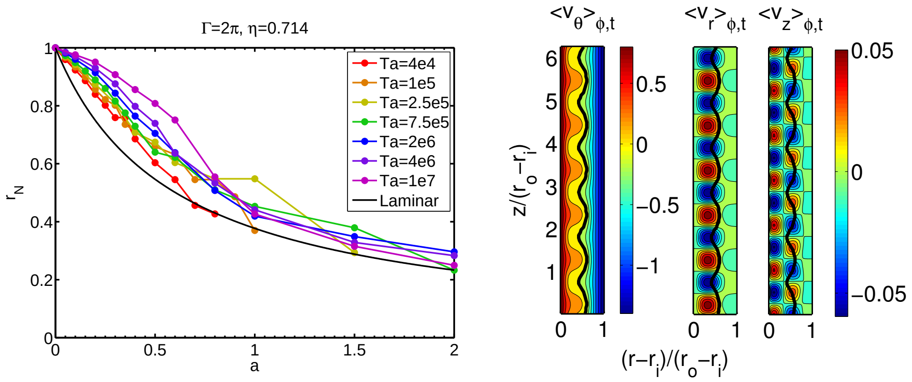


(a) Ta against Nu_ω for $a = 0$ (b) Normalized $(Nu_\omega - 1)$ against a (c) $d\langle\omega'\rangle_{A,t}/dr$ against a in the bulk

Figure 1.

convection is enhanced. The higher the Ta number, the further away this surface is pushed. This is shown in 2(a). If the value of a is high enough, the vortices cannot penetrate the whole domain. This can be seen in Figure 2(b), for $Ta = 7.5 \cdot 10^5$ and $a = 1$.



(a) $\langle r_n \rangle_{A,t}$ against a for $\eta = 0.714$ and several values of Ta (b) Velocities and neutral surface for $Ta = 7.5 \cdot 10^5$ and $a = 1$

Figure 2.

CONCLUSION

A TC system with fixed geometry was simulated. Simulations match with experimental data from [10]. An optimal transport of ω is also revealed, which moves across a as we have not reached the ultimate regime. It is linked to flow parameters already explored in experiments [1].

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