HIGH RAYLEIGH NUMBER CONVECTION IN A LONG VERTICAL TUBE

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Summary Since the theoretical work of Kraichnan [1], the search for asymptotic or 'Ultimate state' has led to several numerical and experimental studies of Rayleigh-Benard convection (RBC). In this regime, the heat transport scaling is expected to be $Nu \sim Ra^{1/2}$, for a fixed Prandtl number (Pr). Although the practical difficulties faced in lab experiments in achieving high Rayleigh Number (Ra) are overcome to a large extent, and the Ra is achieved as high as 10^{17}, there is no consensus on the observation of the ultimate state. In the present study we report results of turbulent convection in an open ended long vertical tube, for the range of Rayleigh numbers (based on gradient, Ra_g) between 5×10^4-5×10^6 and nearly constant Pr~6.5. Above $Ra_g \sim 10^6$, we observe scaling of non-dimensional heat flux as, $Nu \sim (Ra_g.Pr)^{1/2}$. This observation is in accordance with the mixing length model proposed by Arakeri et. al.[2] and subsequent experimental observations of Cholemari & Arakeri [3],[4], where they used salt in water to create the driving density gradient. Below $Ra_g \sim 10^6$, we observe $Nu \sim (Ra_g.Pr)^{0.29}$. This scaling, although is similar to the one observed in RBC, the flux in case of tube convection is larger by an order of magnitude than for RBC, under similar conditions.

INTRODUCTION

The Studies of turbulent Rayleigh Benard convection (Siggia[5], Ahlers et.al.[6], Lohse & Xia[7]) have revealed many global and small scale features of the flow. Most experiments and numerical simulations suggest that in the Nusselt number correlation, $Nu = C.Ra^n$, n is approximately 1/3, which implies that the heat flux is independent of the length scale, the height of the cell. This independence comes from the fact that the flux is essentially controlled by the thin boundary layers at the top and bottom walls. The flow in the long vertical tube becomes axially homogeneous in the absence of these boundary layers and has some different features than the confined RBC cell. The ½ power law is achieved at a lower Ra than predicted for the ultimate state of RBC and the flux for given Ra is an order of magnitude larger than in confined RBC. Experimental work of Gibert et. al.[8] in a vertical channel and numerical simulations of Schmidt et.al.[9], where they replace the top and bottom walls by periodic boundary conditions also report similar observations.

FLOW DESCRIPTION

The turbulent flow is created by unstable density gradient imposed across the ends of a long (length to diameter ratio=10) vertical tube. The flow has some interesting features: it has zero mean flow, it is axially homogeneous in the middle portion of the tube and the turbulent energy production is solely by buoyancy. Since the driving force for the flow is imposed linear density gradient, it is appropriate to define the Rayleigh and Nusselt number based on the gradient. The three non-dimensional numbers which are of importance are, Rayleigh number, Nusselt number and Prandl number, given as,

$$Ra_g = \frac{\beta g (d\overline{T}/dz) d^4}{\nu\alpha}, \quad Nu_g = \frac{q}{k(d\overline{T}/dz)} \quad \text{and} \quad \Pr = \frac{\nu}{\alpha} \quad \text{respectively.}$$

β is cubic expansion coefficient, d is the diameter of the tube, ν and α are kinematic viscosity and thermal diffusivity, q is the average heat flux and k is thermal conductivity. The working fluid is water.

Mixing length model and scaling

According to the mixing length model proposed by Arakeri et.al [1], a fluid particle heavier than the surrounding fluid by an amount ρ′ attains a 'free fall' velocity w_m as it falls through the mixing length d, before it loses its identity in the surrounding fluid. The dimensional analysis gives the scales for density and temperature fluctuations (ρ′, T′), fluctuations of vertical component of velocity (w′) and the average flux as, ρ_m, w_m and F_m respectively.

$$\rho' \sim (d\overline{\rho}/dz)d = \rho_m, \qquad T' \sim (d\overline{T}/dz)d = T_m, \qquad w' \sim \sqrt{\beta g (d\overline{T}/dz) d^2} = w_m,$$

$$\langle flux \rangle = \rho_0 C_p \langle\langle w'T' \rangle\rangle \sim \rho_0 C_p (w_m T_m) = F_m$$

Tube diameter d is considered as the dominating length scale and the two molecular diffusivities, ν and α are not expected to be relevant. Thus from above mixing length scales one can obtain,

$$Nu_g = C_m Ra_g^{1/2} \Pr^{1/2}$$

Experiments

The experiments were done in a set up where a vertical tube interconnected two tanks, each carrying fluid maintained at constant temperature. Tubes of different diameters were used to obtain a wide range of Ra_g. Temperature was measured

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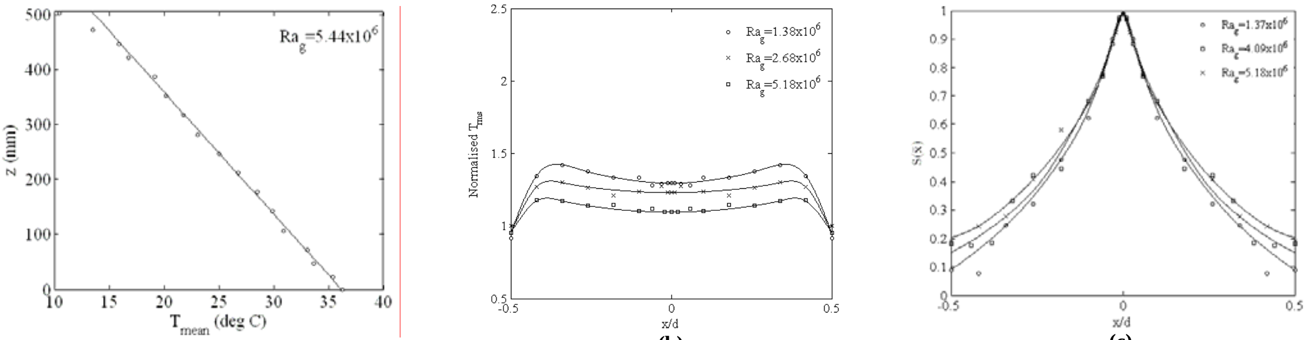
along the tube length by thermocouples and also in the transverse direction at the mid height by fast response thermistors, which were traversed independently to obtain the spatial distribution of T_{rms} and correlations. Average flux was obtained by measuring heat input to the bottom tank.

RESULTS AND DISCUSSION



(a) **(b)** **(c)**

Figure 1. (a) Temperature gradient along the length of the tube, (b) Variation of Trms in the transverse direction, (c) Spatial correlation of temperature fluctuations in the transverse plane for different Ra_g

Figure 1(a) shows the mean temperature gradient along the length of the tube, which was found to be quite linear. The r.m.s. values of temperature fluctuations, non dimensionalised with ΔT over one diameter, across the diameter of the tube are shown in Figure 1(b). Increasing non dimensional T_{rms} for decreasing Ra_g imply that the mixing length scale T_m, applies to higher Ra_g more accurately. The variation is similar as for the r.m.s. values of vertical velocity fluctuations observed by Cholemari & Arakeri [4]. The Spatial correlation function, $S(\bar{r}) = \frac{T'(\bar{x},t)T'(\bar{x}+\bar{r},t)}{Trms(\bar{x}).Trms(\bar{x}+\bar{r})}$, is shown in Figure 1(c), for the transverse tube cross section and different Ra_g. In the core region of x/d of about ±0.1, spatial correlation functions collapse to a single curve and de-correlates faster for decreasing Ra_g, as we move towards tube wall. Figure 2 is the plot of results of the present study, compared with the previous work of Cholemari & Arakeri [4] and results of Schmidt et.al.[9]. The $Nu_g=0.81(Ra_g.Pr)^{1/2}$ scaling is observed for higher Ra_g, while lower Ra_g values show $Nu_g=20.2(Ra_g.Pr)^{0.29}$ scaling. The change in the slope of the fit can be marked at $Ra_g.Pr\sim5\times10^6$. The ½ power law is observed for a range of Pr ($1 \le Pr \le 670$). The transition in the ½ power law seems to be dependent on Pr. Figure 3 is an instantaneous image of the vertical cross section of the middle portion of the tube. The flow is seeded with tracer particles and illuminated with laser light sheet.

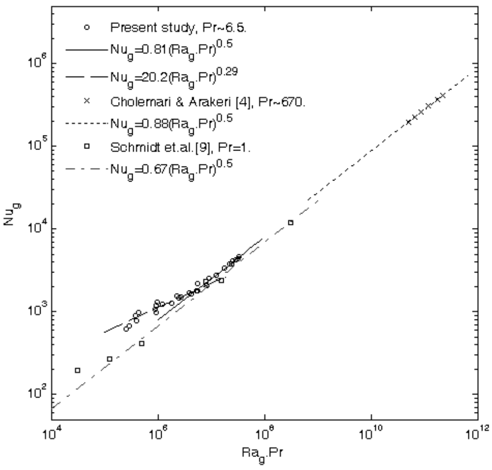


Figure 2. Nu_g scaling with $Ra_g.Pr$

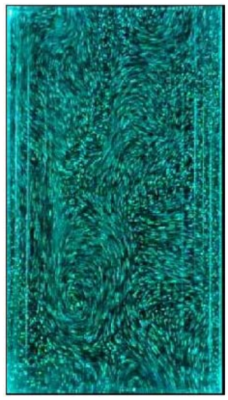


Figure 3. Flow visualization using particles, $Ra_g=1.33\times10^6$

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