UNSTEADY COUPLED THERMAL BOUNDARY LAYERS

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Summary The transition of the flow in a differentially heated partitioned cavity is investigated using numerical simulation. The transition of the flow in the cavity from start-up to an unsteady state is characterized and analyzed. The spectrum of the temperature oscillations in the coupled thermal boundary layers around the partition and the peak frequency are obtained.

INTRODUCTION

Fluid motions may occur on both sides of a vertical wall due to the presence of a temperature difference between the fluids at the two sides of the wall, which is relevant to considerable engineering applications. In previous studies (e.g. [1]), the presence of a thermal boundary layer outside the vertical wall of the cavity is often neglected and the vertical wall is considered isothermal. However, the thermal boundary layers, present on both sides of the vertical wall in real situations, coupled because of heat transfer through the wall. Accordingly, the numerical study, focusing on the coupling effect of thermal boundary layers around a partition, has been performed in [2] in which the transition of the flow in the partitioned cavity to a steady state at the Rayleigh number of $O(10^8)$ is investigated. Nevertheless, the features of the transition of the flow in the partitioned cavity at a higher Rayleigh number from start-up to an unsteady state are unclear, and thus this paper investigates the transition of the flow around the partition to an unsteady state at a Rayleigh number of 5.3×10^{10}.

NUMERICAL PROCEDURES

A two-dimensional (2D) numerical simulation is used to characterize unsteady natural convection flows in the differentially heated partitioned cavity in this study. For convenience, the 2D governing equations are non-dimensionalized using the following scales: x, $y \sim H$; $t \sim H^2/(\kappa Ra^{1/2})$; $(T_h\text{-}T_c) \sim \Delta T$; u, $v \sim \kappa Ra^{1/2}/H$; and $\rho^{-1}\partial p/\partial x$, $\rho^{-1}\partial p/\partial y \sim \kappa^2 Ra/H^3$, and are rewritten in the following non-dimensional forms,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\Pr}{Ra^{1/2}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\Pr}{Ra^{1/2}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\Pr T, \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Ra^{1/2}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right), \quad (4)$$

where x and y are the horizontal and vertical coordinates with origin at the centre of the cavity, t is the time, T is the temperature, T_h and T_c are the temperatures of the hot and cold sidewall, p is the pressure, u and v are the velocity components in the x and y directions, g is the acceleration due to gravity, H is the height of the cavity, ρ is the density, κ is the thermal diffusivity, and ν is the kinematic viscosity. The three dimensionless parameters which govern the flow are the Rayleigh number (Ra), the Prandtl number (Pr) and the aspect ratio (A), defined as

$$Ra=\frac{g\beta(T_h-T_c)H^3}{\nu\kappa},\ Pr=\frac{\nu}{\kappa},\ A=\frac{H}{L}, \quad (5)$$

where L is the width of the cavity and β is the coefficient of thermal expansion. Note that the working fluid in the cavity is water with a Prandtl number of $Pr = 7.8$, the aspect ratio of the cavity is unity ($A = 1$, see Fig. 1), and the Rayleigh number is $Ra = 5.3 \times 10^{10}$ in this study. Initially, the temperatures of the motionless fluids at the left and right sides of the partition are T_c and T_h, respectively. The top and bottom walls are adiabatic; the two sidewalls are isothermal at T_c and T_h respectively; the partition of zero thickness is diathermal for which only horizontal heat transfer is considered; and all interior walls and the partition are rigid and no-slip.

The governing equations are implicitly solved using a finite-volume SIMPLE algorithm [3]. A hybrid grid system with finer non-uniform grids concentrated in the proximity of all boundaries and relatively coarse uniform grid in the interior region is constructed. Grid dependence tests have been carried out and the grid system of 403×199 is adopted. Further, time-step-dependence tests have been performed, which suggests that a time-step of 0.0013 may be applied in the present study.

NUMERICAL RESULTS

Fig. 1 shows the unsteady flow at a large time. Isotherms in Fig. 1(a) reveal distinct waves around the partition. Further, it is clear that waves become stronger as they travel downstream at both sides of the partition, which is also confirmed by streamlines shown in Fig. 1(b).

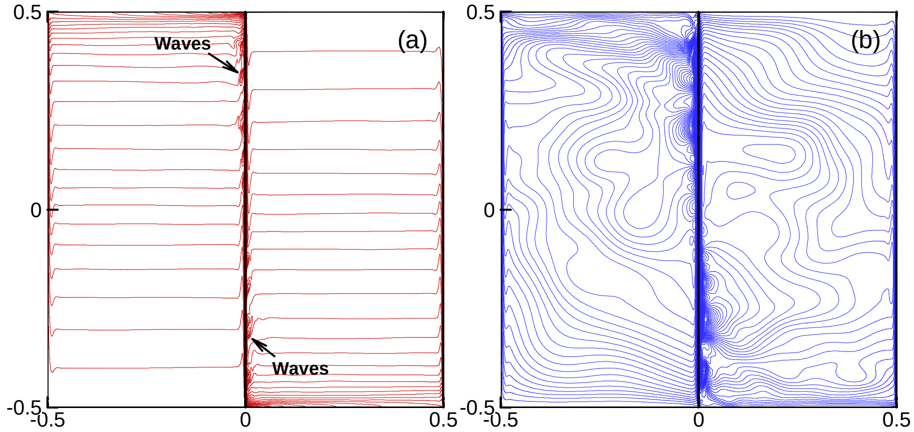

FIG.1. Flows in the partitioned cavity at t = 258. (a) Isotherms with an interval of 0.025. (b) Streamlines with an interval of 0.0015.

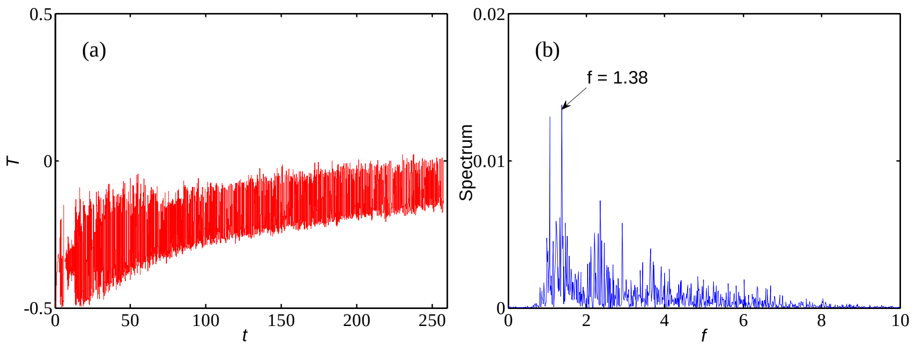

FIG.2. Temperature and spectrum at the point (x = -0.04, y = 0.4) in the downstream thermal boundary layer on the left side of the partition. (a) Temperature. (b) Spectrum.

For the purpose of observing further the transition of the flow around the partition to an unsteady state, a time series of the temperature at the point in the downstream thermal boundary layer at the left side of the partition is plotted in Fig. 2(a). It is seen that the temperature is oscillatory at a large time; that is, the transition of the flow in the partitioned cavity approaches to an unsteady (periodic) state at the present Rayleigh number. The further spectrum analysis shows that the peak frequency is f = 1.38.

CONCLUSIONS

The transition of the flow in the partitioned cavity to an unsteady state is numerically investigated. The unsteady flow around the partition is observed, and the oscillations in the coupled thermal boundary layer around the partition are analyzed. The spectrum of the temperature in the coupled thermal boundary layer and the peak frequency are obtained. The further study and the comparison with the corresponding experiments are being still carried out.

Acknowledgements

This research was funded by National Natural Science Foundation of China (Project: 11142015) and the Fundamental Research Funds for the Central Universities (Project: 2011JBM360).

References

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