LAGRANGIAN MEASUREMENTS IN TURBULENT THERMAL CONVECTION

Ke-Qing Xia, Rui Ni, & Shi-Di Huang
Department of Physics, The Chinese University of Hong Kong, Shatin, Hong Kong, China

Summary The Lagrangian and Eulerian statistics in turbulent Rayleigh-Bénard convection are investigated by particle tracking velocimetry (PTV) method. The study covered a wide range of Rayleigh number Ra and in two convection cells with different sizes. The present study confirms previous findings in two-dimensional Eulerian velocity measurements and gives a better resolution to the dissipative range structure function. Moreover, the longitudinal and transverse second-order structure functions are obtained and their relationship in inertial range confirms that in the center of convection cell the flow is close to homogeneous and isotropic turbulence.

Introduction

The turbulent motion in a fluid layer heated from below and cooled from above, i.e. Rayleigh-Bénard (RB) convection [1, 2], is a fruitful model system for the understanding of buoyancy-driven turbulence, which is ubiquitous both in nature and industry. The turbulent flow in the convection cell could be characterized in terms of points fixed in space or in terms of the trajectory of fluid element, which are usually refered to as the Eulerian frame and Lagrangian frame respectively. The Lagrangian studies of turbulence become feasible in recent years thanks to the progress made in particle tracking (PTV) method, and the results obtained by PTV method could be analyzed both in Eulerian and Lagrangian frames. An important issue in the study of turbulent convection is to find the universal properties of small-scale turbulence. One well-known quantity to characterize these properties is the Eulerian second-order velociy structure functions (ESF2). Defining the Eulerian velocity increment as $\delta\mathbf{u}(\mathbf{r}) = \mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})$ with $\mathbf{r}$ being the separation between two velocity vectors, the ESF2 is given by $D_{ij} = \langle \delta u_i(\mathbf{r})\delta u_j(\mathbf{r})\rangle$. In homogeneous and isotropic turbulence (HIT), the ESF2 could be separated into two components. One is for the velocity increment along the direction of separation and the other one is for the one perpendicular to the direction of separation, which is so-called the longitudinal ($D_{LL}(r)$) and transverse ($D_{NN}(r)$) Eulerian second-order structure functions, respectively. Kolmogorov proposed that for HIT there are three different regimes of turbulence [3]. At the large spatial and temporal scales, the energy is transfered from the forcing force to the turbulent flow. For these scales, the statistics is dominated by the specific forcing mechanism, which is, in our case, the large-scale circulation driven by thermal plumes. For the small scales, the molecular viscosity becomes dominant. And between the large and small scales, there is a range of intermediate scales. Kolmogorov suggested that the turbulent statistics in this range should have a universal form independent either of the large-scale flow or of the molecular viscosity, and the only relevant parameter is the turbulent energy dissipation rate, which gives a prediction to the scaling law of ESF2, i.e. $D_{LL},\ D_{NN} \sim r^{2/3}$ (K41 scaling). This intermediate range is known as the inertial range, and there is a length scale that separate the inertial range and those smaller scales, which is Kolmogorov scale η. For those small scales much less than η, it is so-called dissipative range, and one expect that $D_{LL},\ D_{NN} \sim r^2$. In addition, Bolgiano and Obukho proposed that for stably stratified convection there is a length scale l_B that divides the cascades of turbulent velocity and temperature fluctuations in the inertial range into two different dynamics regimes dominated by buoyancy force (large than l_B) and inertial force (smaller than l_B) [4, 5]. For the regime dominated by buoyancy force, the scaling for ESF2 is predicted to be $D_{LL},\ D_{NN} \sim r^{6/5}$ (BO59 scaling). Previous experiments in turbulent convection Eulerian structure function used the two-dimensional particle image velocimetry [6, 7], which could only measured the Eulerian velocity structure functions with two velocity components with the turbulent flow being three-dimensional. The present study aims to measure the three-dimensional ESF2 covering both dissipative and inertial ranges in the center of turbulent convection cell and comparing our experimental results with the theoretical predictions.

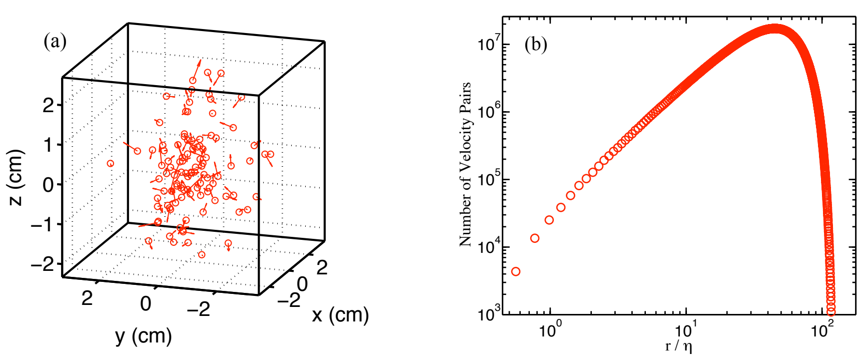

Figure 1: (a) One snapshot of the Eulerian velocity field for randomly-chosen one hundred particles at $Ra = 1.3 \times 10^{10}$ and $Pr = 4.4$. The circles show the particle positions, and the arrows represent the velocity of that particle with the length of arrows indicating the magnitude ranging from 1 mm/s to 2 cm/s. (b) The number of velocity pairs as a function of the separation between the two for $Ra = 1.3 \times 10^{10}$ and $Pr = 4.4$.

[a] Corresponding author. E-mail: kxia@phy.cuhk.edu.hk.

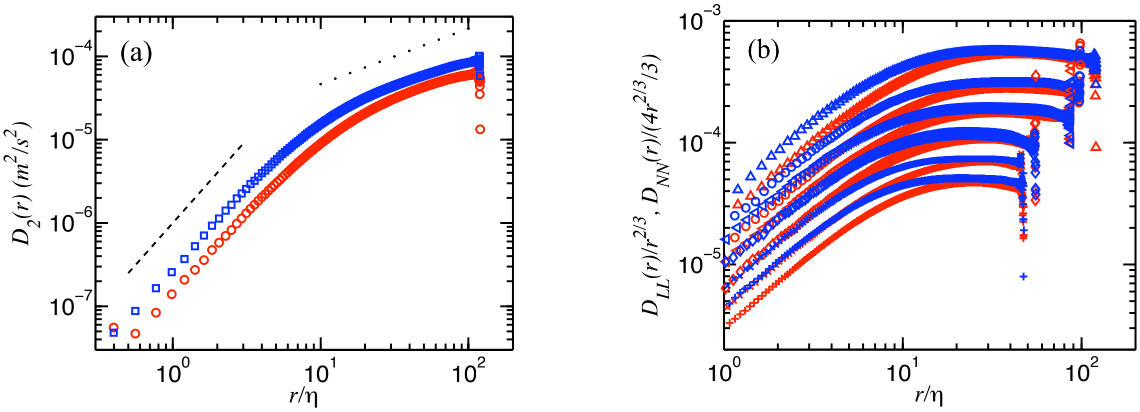

Figure 2: (a) The Eulerian second-order structure functions for different separations for $Ra = 1.3 \times 10^{10}$ and $Pr = 4.4$. The blue squares and red circles represent the transverse $D_{NN}(r)$ and longitudinal components $D_{LL}(r)$, respectively. The dashed and dotted lines show the dissipative range and inertial range scaling with respective exponents being 2 and 2/3. (b) Longitudinal ($D_{LL}(r)$) and transverse ($D_{NN}(r)$) Eulerian second-order structure functions compensated by $r^{2/3}$ and $4r^{2/3}/3$, respectively. From bottom to top, six pairs of curves are for $Ra = 1.3 \times 10^9,\ 2.0 \times 10^9,\ 3.0 \times 10^9, 6.1 \times 10^9,\ 7.8 \times 10^9$, and 1.3×10^{10}.

Experimental setup and procedures

The experiments were carried out in convection cells using water as working fluid. The height and inner diameter of each convection cell are the same, and the respective heights for two cells are 19.2 cm (small cell) and 48.6 cm (large cell). The upper and lower plates are made of copper and the sidewall is made of transparent Plexiglas. The tracking volume for two cells are approximate (5 cm)^3 and (11 cm)^3, which is illuminated by an expanded laser beam. There are three fast cameras focusing from different angles to the same volume, and the tracers we used is polyamid particles with 50 μm diameter and density 1.03 g/cm^3. The position error after calibration is roughly 8 μm with the minimum resolvable separation between one pair of particles $100 \sim 200$ μm because of the particle size and diffraction effect. The number of seeding particles in the view volume is typically 500. Other details of the setup has been described elsewhere [8].

Results and discussion

One snapshot for the instantaneous Eulerian velocity field analyzed from the Lagrangian tracking results is shown in Fig. 1 (a). There are roughly 500 particles in the volume, but we only show 100 particles for clarity. It is seen that both the positions of velocity vectors and the separation between them are random. We manually choose the bin size for the separation to be $\Delta r = \eta/4$, and show the number of velocity pairs for different separation in Fig. 1 (b). It is noted that for all interested length scales the number of velocity pairs are larger than 5×10^3, and there is a most probable separation r_m between two velocity vectors, which is roughly 2.2 cm. The number drops very fast for $r > r_m$ because of the finite size of the view volume, which could explain the scatter of the data for $r > 100\eta$ in Fig. 2 (a). One example of ESF2 results obtained by PTV method is shown in Fig. 2 (a). Comparing with the K41 scalings in the dissipative and inertial ranges indicated by dashed and dotted lines in the figure, it is evident that the data for both two components agree with the K41 predictions very well, and there is no clear range with BO59 scaling. This is consistent with the previous findings in two-dimensional velocity measurements [6]. Ni *et al.* have showed that $D_{NN}(r) = 2D_{LL}(r)$ in the dissipative range, which indicates the flow follows the local isotropy assumption in that range [9]. Here, to further confirm the relationship between D_{LL} and D_{NN}, the two structure functions compensated by the inertial range scaling were shown in Fig. 2 (b). Since in the inertial range $D_{NN}(r) = 4D_{LL}(r)/3$ for isotropic turbulence, the D_{LL} was compared with D_{NN} divided by the prefactor 4/3. If the relation holds, the two curves for one Ra should collapse together in the inertial range. It is seen that there is at least a short range for two curves overlap with each other for each Ra, and the plateau is just at the collapse positions between the two curves, which indicates the flow in the center of convection cell is isotropic turbulence in the inertial ranges as well.

This work is supported by Hong Kong Research Grants Council under Grant No. CUHK404808, 404409, 403811.

References

[1] G. Ahlers, S. Grossmann, and D. Lohse, Rev. Mod. Phys. **81**, 503 (2009).
[2] D. Lohse and K.-Q. Xia, Annu. Rev. Fluid Mech. **42**, 335 (2010).
[3] A. N. Kolmogorov, Dokl. Akad. Nauk SSSR **30**, 299 (1941).
[4] A. M. Obukhov, Dokl. Akad. Nauk SSSR **125**, 1246 (1959).
[5] R. Bolgiano, J. Geophys. Res. **64**, 2226 (1959).
[6] C. Sun, Q. Zhou, and K.-Q. Xia, Phys. Rev. Lett. **97**, 144504 (2006).
[7] Q. Zhou, C. Sun, and K.-Q. Xia, J. Fluid Mech. **598**, 361 (2008).
[8] R. Ni, S.-D. Huang, and K.-Q. Xia, J. Fluid Mech. (2011), accepted.
[9] R. Ni, S.-D. Huang, and K.-Q. Xia, Phys. Rev. Lett. **107**, 174503 (2011).