VORTEX MOTIONS IN SOFT AND HARD 2-D TURBULENT RAYEIGH-BÉNARD CONVECTION[a)]

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Summary The two-dimensional Rayleigh-Bénard heat convection is studied by direct numerical simulations (DNS) with a wide range of Rayleigh numbers ($Ra = 5 \times 10^7 \sim 8 \times 10^{11}$, $Pr = 0.7$, $\Gamma = 1$). Two types of scaling of the Nusselt number Nu with the Rayleigh number Ra, $Nu \propto Ra^\alpha$, are found at $Ra_c \approx 6 \times 10^8$, with $\alpha \approx 0.33$ before and $\alpha \approx 0.284$ after the transition. We interpret them as a transition from *soft* to *hard* turbulence, and find a key support to this interpretation from the dynamics of the vortex motion. At small Ra, the large scale circulation (LSC) and the corner vortices (CV) are stable and the heat flux is steadily transported from the heat wall (and the cold wall) by thermal plumes generated by LSC and CV. At $Ra \geq 6 \times 10^8$, the CV detach intermittently from the corner and entrained to the bulk flow. We propose that this detachment introduces strong fluctuations in temperature field, and is responsible for the new regime of heat transfer, determined by a mixture of thermal plumes, detached CV and eddies.

INTRODUCTION

Rayleigh-Bénard convection (RBC), the buoyancy driven flow in a fluid layer heated from below, is one of the fundamental physical problems for geophysical, astrophysical, and engineering applications. The flow is characterized by Rayleigh number Ra, a ratio of driving due to buoyancy resulting from a vertical temperature gradient to damping due to the fluid's viscosity and thermal diffusion, and Prandtl number $Pr = \nu/\kappa$, a ratio of the fluid's kinematic viscosity to its thermal diffusivity. The vertical heat transport by the convective flow is measured by the Nusselt number Nu. One of the goals in many studies is to quantify Nu as a function of Ra and Pr and the geometry (often aspect ratio Γ) of the apparatus. For fixed Prandtl number and aspect ratio, the Rayleigh number dependence often displays power laws: $Nu \sim Ra^\alpha$. The early theory is Malkus' marginal-stability theory of 1954. It assumed that the thickness of the thermal boundary layer adjusts itself so as to yield a constant (critical) Rayleigh number. This predicts $\alpha = 1/3$ [1]. Later, Castaing *et al.* suggest a smaller power-law exponent with a mixing-zone model, which yielded $\alpha = 0.286$ [2]. However, there is still no universally accepted explanation for how the flow pattern effects the Nu-Ra relationship. The present study investigates the transition from one regime to another with the notion of soft to hard turbulence, and with a connection to flow dynamics pattern.

NUMERICAL SIMULATION

The governing momentum, energy and continuity equations for the Rayleigh-Bénard problem in 2D cell coordinates (x, z) are solved by employing the SIMPLE algorithm [3]. The dimensionless temperature varies between $T|_{z=0} = +0.5$ at the bottom and $T|_{z=H} = -0.5$ at the top horizontal walls and satisfies $\partial T/\partial x = 0$ on the vertical walls. We set $Pr = 0.7$ and the aspect ratio equals $\Gamma = 1$; the Rayleigh number varies from 5×10^7 to 8×10^{11}. Staggered structured non-equidistant grids were used in the simulations consists of up to 4096×2500 (for $Ra = 8 \times 10^{11}$) nodes in the vertical and horizontal directions, respectively.

TRANSITION OF FLOW PATTERNS

Figure 1(a) is the measured Nu versus Ra, which yields two distinct power-laws across $Ra_c \approx 6 \times 10^8$. For $Re < Ra_c$, the power-law exponent is $\alpha = 0.33$, close to Malkus's prediction. When $Ra \geq Ra_c$, however, the measured power-law exponent is close to 0.284, very close to the mixing-zone model's prediction $\alpha = 2/7$. In order to examine the nature of flow dynamics associated with the two regimes, we set several probes near the heated wall and recorded the time sequence of velocity $\overrightarrow{u}$ and temperature T at these positions. Figure 1(b) and 1(c) show the horizontal velocities at two points near the wall for $Ra = 3 \times 10^8$ and 1×10^{10}, respectively. The quasi-periodicity of the horizontal velocity fluctuations, as shown in Figure 1(b), demonstrates a chaotic flow pattern before the transition. On the other hand, considerable turbulent time signals are seen after the transition, as shown in Figure 1(c). We propose that the former correspond to a soft turbulence regime and the latter to a hard turbulence regime [4]. In the latter, small-scale fluctuations are generated, as a typical turbulent state must exhibit.

Figure 2 is the visualization of the instantaneous streamlines and the temperature field. The relatively steady large-scale circulation exists for Rayleigh numbers simulated. However, the CV are sensitive to Ra. At lower Ra before the transition, the CV are firmly trapped in the cell's corners (see Figure 2(a)). When $Ra \geq Ra_c$, the CV no longer stays in the corner region, but continuously detach from the corner and regenerate (see Figure 2(b)). The detached CV are entrained to the LSC, together with the thermal plumes, determine a new regime for the bulk flow. Therefore, after the transition, the three

[a)] The project is supported by the National Natural Science Foundation of China (10572004, 90716008, 10921202, 11172006), MOST 973 Project (2009CB724100).
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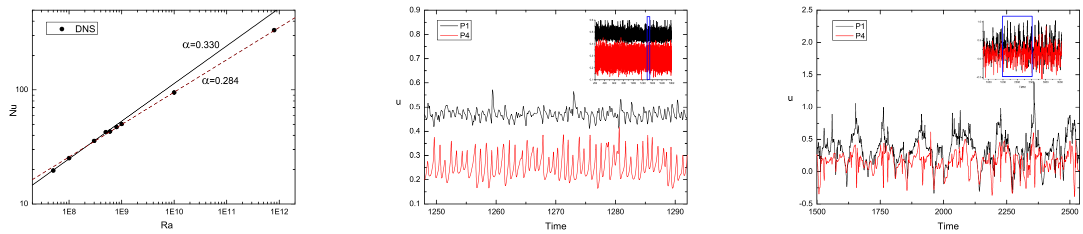
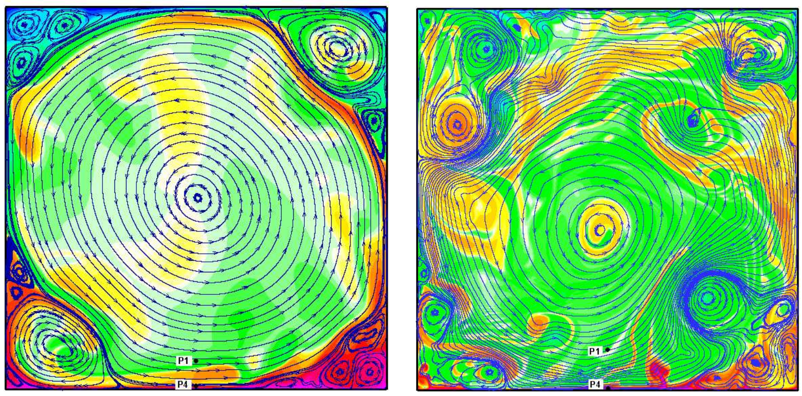


(a) Nu-Ra (b) $Ra = 3 \times 10^8$ (c) $Ra = 1 \times 10^{10}$

Figure 1. (a) The steady-state Nusselt number as function of the Rayleigh number in 2D simulations. Two power-laws are identified by the critical $Ra_c = 6 \times 10^8$. (b) and (c) are the horizontal velocity signals at two fixed positions near the bottom wall. Probe position $P1$ is farther from the wall than $P1$, which is marked in Figure 2. The insets are the signals with a long record time. The blue frames indicates the time sequences presented in the main figures.

elements (LSC, CV and thermal plumes) jointly play a role in the heat transfer. We calculated the energy dissipation rate ϵ_u and the thermal dissipation rate ϵ_θ in the middle region of the cell ($H = 0.5$). The flow before transition is characterized by strong dissipation rate near the boundaries. For $Ra = 5 \times 10^7$, 91% of the total ϵ_u is occupied by near-wall flows ($0 \leq x \leq 0.1$), but for $Ra = 6 \times 10^8$, only 72% of the total ϵ_u is at the same region. ϵ_θ in contrast to ϵ_u is more sensitive to Ra. For $Ra = 5 \times 10^7$, 99% of the total ϵ_θ is limited in the near wall region ($0 \leq x \leq 0.138$), but for $Ra = 6 \times 10^8$, only 50% of the total ϵ_θ is in the same region. An interesting question is: whether is the CV the origin of the mixing zone suggested by Castaing *et al.* [2]?



(a) $Ra = 3 \times 10^8$ (b) $Ra = 1 \times 10^{10}$

Figure 2. The instantaneous flow field of temperature (the colored clouds) and stream lines (the blue lines). The temperature inside the thermal plumes is higher than that of the background fluid. $P1$ and $P4$ are the probe positions recording the velocity signals.

CONCLUSIONS

The soft and hard turbulence regime of 2D Rayleigh-Bénard heat convection are studied. A scaling transition of power law exponent from 0.33 to 0.284 is discovered at $Ra_c = 6 \times 10^8$. It is shown that velocity fluctuation signals is chaotic before and turbulent after the transition. An particular, it is shown that CV is the stable before the transition, unstable and detached from the corner after the transition. It is then proposed that turbulent fluctuations are responsible for the scaling 0.284.

References

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