SCALING OF THE LOCALLY AVERAGED THERMAL DISSIPATION RATE IN TURBULENT RAYLEIGH-BÉNARD CONVECTION

Penger Tong*[a)], Xiaozhou He[*] & Emily S. C. Ching [**]
[*]*Department of Physics, Hong Kong University of Science and Technology*
Clear Water Bay, Kowloon, Hong Kong, China
[**]*Department of Physics and Institute of Theoretical Physics*
The Chinese University of Hong Kong, Shatin, Hong Kong, China

Summary Using a homemade local temperature gradient probe, the instantaneous thermal dissipation rate $\epsilon_T(\mathbf{x},t)$ was obtained in an aspect-ratio-one cylindrical convection cell filled with water. From the time series measurements, we construct a locally averaged thermal dissipation $\epsilon_T(\tau)$ over a time interval τ and systematically study its statistics and scaling properties. It is found that the moments of $\epsilon_T(\tau)$ exhibit good scaling in τ, i.e., $<[\epsilon_T(\tau)]^p>\sim\tau^{\mu(p)}$, for all p up to 6. The obtained exponent $\mu(p)$ at two representative locations in the convection cell is well explained by a phenomenological model, which combines the effects of velocity statistics and geometric shape of the most dissipative structures in turbulent convection.

Introduction
To test the refined similarity ideas for the anomalous scaling of passive scalars in a turbulent flow, one not only needs to check the scaling properties of the velocity and temperature structure functions, but also should examine the scale-dependent statistics of the dissipation fields, in order to verify that the observed anomalous scaling in the velocity and temperature structure functions is indeed compatible with the scaling of the dissipation fields. In contrast to the large number of experimental studies of the velocity and temperature structure functions, direct measurements of the viscous and thermal dissipation rates in turbulent flows are rare [1,2]. This is partially due to the fact that simultaneous measurements of all the components of the velocity gradient tensor or the temperature gradient vector with adequate spatial and temporal resolutions are still challenging tasks.

In this paper, we report a systematic study of the scaling properties of the measured local thermal dissipation rate $\epsilon_T(\mathbf{x},t)$ in turbulent Rayleigh-Bénard convection. To address issues related to the flow anisotropy and inhomogeneity, herein we decompose the locally-averaged thermal dissipation rate $\epsilon_T(\tau)$ over a time interval τ into three terms, $\epsilon_T(\tau) = \mathrm{sum}_i\, \epsilon^i_T(\tau)$ (i = x, y, z); each term results from a component of the temperature gradient vector in the corresponding direction. We focus our attention on the scaling properties of each contribution $\epsilon^i_T(\tau)$ at two representative locations in the cell: at the cell center far away from the boundaries and near the sidewall at mid-height of the cell. In the latter location, the flow field is inevitably influenced by the large-scale circulation that spans the height of the convection cell and by the thermal plumes erupted from the lower thermal boundary layer. Such a systematic study allows us to disentangle the intermittency effects from the flow anisotropy and inhomogeneity and thus to have a critical test on the theories of anomalous scaling.

Experiment
The experiment was conducted in an upright cylindrical convection cell filled with water. The inner diameter of the cell is D = 19.0 cm and its height H = 20.5 cm, the corresponding aspect ratio of the cell is $\Gamma = D/H \approx 1$. Details about the apparatus and experimental method have been described elsewhere [3]. The temperature gradient probe was made of four small thermistor beads of 0.11 mm in diameter and was assembled in our own lab. The four identical thermistors are used to measure the three components of the local temperature gradient simultaneously. One of the thermistors is placed at the origin, labelled as T_0, and the other three thermistors are arranged along the x-, y-, and z-axis, respectively. By simultaneously measuring the four temperature signals, we obtain the three temperature gradient components $\delta T_i/\delta l$, where $\delta T_i = T_i - T_0$ (i = x, y, z) is the temperature difference between a pair of the thermistors with separation $\delta l = 0.25$ mm. All the thermistors were calibrated individually with an accuracy of 5 mK for δT_i.

Experimental Results
We first discuss the scaling behavior of $\epsilon^z_T(\tau)$ near the sidewall (at the middle height of the cell and 1~cm away from the cell wall). Figure 1 shows the compensated plot of the moments, $<[\epsilon^z_T(\tau)]^p>/[(\epsilon^z_0)^p\tau^{\mu z(p)}]$, as a function of τ for p = 2, 4 and 6 (from bottom to top). A common scaling (flat) region in τ is found for all the values of p, suggesting that the vertical moment, $<[\epsilon^z_T(\tau)]^p>/(\epsilon^z_0)^p$, has a good power-law scaling on τ. The scaling range in τ is between 1 s and 10 s, with the lower bound remained the same as that in the central region. The upper bound of the scaling range is reduced by a factor of two when compared with that at the cell center, because the large-scale fluctuations are cut off by the nearby cell wall. Similarly good power-law scaling is also found for the two horizontal moments, $<[\epsilon^x_T(\tau)]^p>/(\epsilon^x_0)^p$ and $<[\epsilon^y_T(\tau)]^p>/(\epsilon^y_0)^p$ (not shown). Using a fixed scaling region in τ (which is chosen from the p=6 curve), we fit all the data to a power law and find the exponents, $\mu^i(p)$ (i = x, y, z), for all values of p up to 6.

[a)] Corresponding author. Email: penger@ust.hk

Figure 2 compares the three scaling exponents, $\mu^x(p)$ (circles), $\mu^y(p)$ (triangles) and $\mu^z(p)$ (diamonds), as a function of p. The scaling exponents in the two horizontal directions overlap very well and can be well described by the equation

$$\mu^i(p) = c(1-\beta^p) - \lambda p, \qquad (1)$$

where c, β and λ are the parameters characterizing the velocity statistics and geometric shape of the most dissipative structures. The following values of c, β and λ give the best fit:

$$c = 1, \quad \beta = \frac{1}{3}, \quad \lambda = \frac{2}{3}. \quad \text{(central region)} \qquad (2)$$

The same set of parameters c, β and λ are also found to fit the obtained values of $\mu^i(p)$ (i = x, y, z) in the central region of the cell (dashed line). Figure 2 thus demonstrates that the horizontal exponents, $\mu^i(p)$ (i = x, y), near the sidewall remain the same as those at the cell center. As discussed in [4], the value c = 1 suggests that the most dissipative structures at the cell center are two-dimensional and sheetlike. The value λ = 2/3 indicates that the velocity fluctuations in the central region obey the Kolmogorov scaling for non-buoyant flows. This result thus suggests that in the central region, buoyancy effects are weak and do not affect the velocity statistics very much. In this sense, temperature behaves like a passive scalar.

The vertical exponent $\mu^z(p)$, however, is different from the horizontal exponents and can be described by the equation Eq. (1) with a different set of parameters, c, β and λ, given below (solid line):

$$c = 2, \quad \beta = \frac{2}{3}, \quad \lambda = \frac{2}{3}. \quad \text{(vertical component in the sidewall region)} \qquad (3)$$

It has been shown [5] that the value of λ is determined by the strength of buoyancy effects, which should only depend on the height of the measuring position. This is indeed observed with the value of λ = 2/3 obtained in the sidewall region (at mid-height of the cell) remaining the same as that at the cell center. On the other hand, the value of c changes with the orientation with the vertical exponent being different from the horizontal ones. For the vertical exponent near the sidewall, we find c = 2, suggesting that the most dissipative structures along the vertical direction are filament-like.

Our experiment clearly demonstrates that the thermal dissipation field in turbulent convection indeed has the scale-dependent statistics and its scaling in τ is well described by a set of scaling parameters. Such a set of parameters provide a scaling description of flow characteristics, including passive or active temperature fluctuations and geometric shape of the most dissipative structures. The intermittency problem of passive scalars has been studied and understood in the Kraichnan model. However, a quantitative experimental study of the correlation between the scaling exponents for the dissipation fluctuations and those for the temperature (passive or active) and velocity structure functions remains to be done. Evidently, the present work represents an important first step toward this direction, particularly in the context of turbulent thermal convection.

Acknowledgements
This work was supported by the Hong Kong Research Grants Council under Grant No. HKUST-603208 (PT) and CUHK-400708 (ESCC).

References

[1] Lohse D. and Xia K.-Q.: Small-scale properties of turbulent Rayleigh-Benard convection. Annu. Rev. Fluid Mech. **42**, 335 (2010).
[2] Monin A. S. and Yaglom A. M., Statistical Fluid Mechanics (MIT Press, Cambridge, MA, 1975).
[3] He X.-Z. and Tong P.,: Measurements of the thermal dissipation field in turbulent Rayleigh-Benard convection. Phys. Rev. E **79**, 026306 (2009).
[4] He X.-Z., Ching E. S. C., and Tong P.: Locally-averaged thermal dissipation rate in turbulent thermal convection: A decomposition into contributions from different temperature gradient components. Phys. Fluids **23**, 025106 (2011).
[5] He X.-Z., Tong P., and Ching E. S. C.: Statistics of the locally-averaged thermal dissipation rate in turbulent Rayleigh-Benard convection. J. Turbulence, **11**, No. 35 (2010).

Figure 1

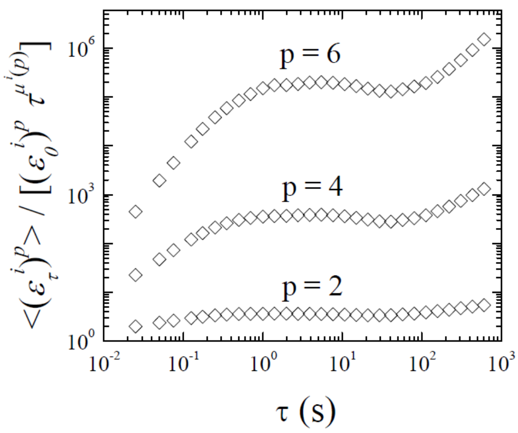

Figure 2