EQUATIONS FOR CONVECTION IN BINARY MIXTURE: SYMMETRY ANALYSIS AND EXACT SOLUTIONS

Viktor K. Andreev*, Irina V. Stepanova*[a)]
*Institute of Computational Modelling, Siberian Branch of Russian Academy of Sciences, Krasnoyarsk, 660036, Russia

Summary Group classification problem for thermodiffusion equations is solved corresponding with arbitrary function which defines buoyancy force. Different specializations of qualified function and generators admitted by the governing equations depending on the values of the function are found. The invariant solutions are constructed and applied to describing thermodiffusion convection in vertical flat layers.

INTRODUCTION

This paper deals with the symmetry analysis and exact solutions of equations describing thermodiffusion convection in a binary mixture. Thermodiffusion (Soret effect) is a molecular transport of substance associated with a thermal gradient [1]. It results in component separation in non-uniformly heated fluid. Mixture density is an arbitrary function of temperature and concentration. Mathematical model of this process is described by Navier-Stokes equations and mass and heat transport equations and is written in form

$$\begin{aligned} &\boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\rho_0^{-1}\nabla p + \nu\nabla^2\boldsymbol{u} + F(T,C)\,\mathbf{g},\\ &T_t + \boldsymbol{u}\cdot\nabla T = \chi\nabla^2 T,\\ &C_t + \boldsymbol{u}\cdot\nabla C = D\nabla^2 C + D_T\nabla^2 T,\\ &\nabla\cdot\boldsymbol{u} = 0, \end{aligned} \tag{1}$$

here $\boldsymbol{x} = (x^1, x^2, x^3)$ is the coordinate vector, $\boldsymbol{u} = (u^1, u^2, u^3)$ is the velocity vector, p is the pressure, T and C are the deviations of temperature and concentration of light component from their mean constant values, ρ_0 = const is the mixture density at the mean values of temperature and concentration, $\mathbf{g} = (0, 0, -g)$ is the gravitational acceleration vector, ν is the kinematic viscosity, χ is the thermal diffusivity, D is the diffusion coefficient, D_T is the thermodiffusion coefficient, $F(T,C)$ is the arbitrary positive function defining buoyancy force. The case $D_T < 0$ corresponds to positive Soret effect when the lighter component is driven towards the higher temperature region. In case of negative Soret effect we have $D_T > 0$ and the opposite situation is observed.

SYMMETRY PROPERTIES OF THE GOVERNING EQUATIONS

Symmetry analysis provides a powerful tool for studying differential equations. This method is very fruitful in application to the equations of physics and mechanics since many of them are derived on the basis of invariance principles [2]. In Lie theory each Lie group transformation is put in correspondence to Lie algebra of differential generators. Using of symmetry analysis methods to study concrete models of mathematical physics consists of calculation of admitted algebra of differential generators depending on the form of arbitrary functions, construction of the optimal system of subalgebras and obtaining of classes of invariant solutions. After a considerable amount of calculations it is shown that equations (1) admit the basic Lie algebra of generators for arbitrary function $F(T,C)$:

$$\begin{aligned} &X_0 = \partial_t,\ X_{12} = x^1\frac{\partial}{\partial x^2} - x^2\frac{\partial}{\partial x^1} + u^1\frac{\partial}{\partial u^2} - u^2\frac{\partial}{\partial u^1},\\ &H_i(h^i(t)) = h_i(t)\frac{\partial}{\partial x^i} + h^i_t\frac{\partial}{\partial u^i} - x^i h_{tt}\frac{\partial}{\partial p},\ \ i = 1,2,3,\ \ H_0(\varphi(t)) = \varphi(t)\frac{\partial}{\partial t}, \end{aligned} \tag{2}$$

where $h^i(t),\ \varphi(t)$ are the arbitrary smooth functions. Also 43 specializations of arbitrary function $F(T,C)$ and corresponding Lie algebras of generators are calculated. These generators can be used for obtaining exact analytical solutions and for reducing the number of dependent and/or independent variables in case of different forms of function defining buoyancy force.

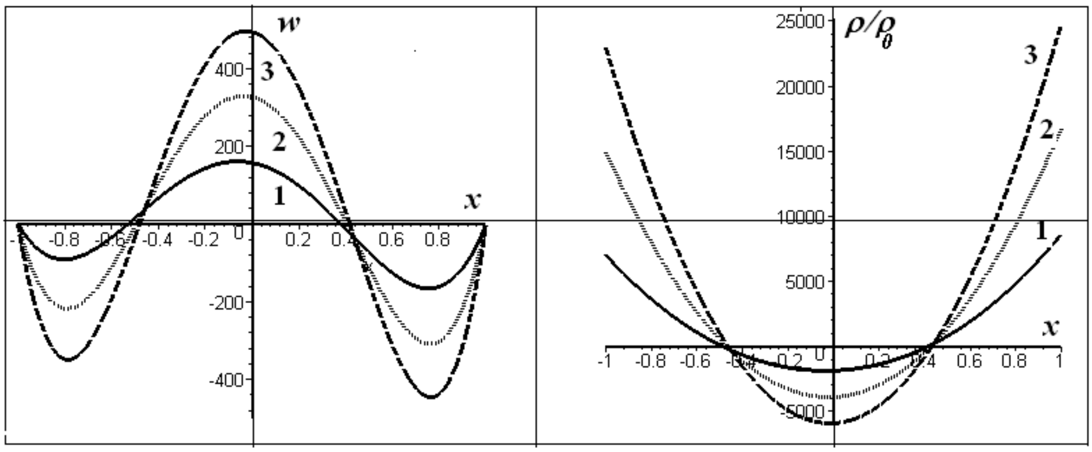
BOUNDARY VALUE PROBLEMS FOR THERMODIFFUSION EQUATIONS

We consider examples of the invariant solutions constructed for some subalgebras. The use of such subalgebras reduces integration of the constructive system to solution of ordinary differential equations.

Example 1 We find an invariant solution of system (1) with respect to the generators

$$\partial_t,\ \partial_{x^2},\ \partial_{x^3} + \varphi\partial p + A\partial_T + B\partial_C, \tag{3}$$

[a)] Corresponding author. E-mail: stepiv@icm.krasn.ru.



Pic. The velocity and density profiles for different T_0: 1—$T_0 = 100$; 2 — $T_0 = 200$; 3 – $T_0 = 300$. Values of parameters: Lewis number $\mathrm{Le} = 100$, Galileo number $\mathrm{Ga} = 100$, temperature difference on the walls $\overline{\theta} = \pm 10$.

here $\varphi,\ A,\ B$ are the constant gradients of pressure, temperature and concentration in x^3-direction. The buoyancy force is presented by function $F(T, C) = f(\frac{2D_T}{\chi - D}T - C)$, here f is the arbitrary smooth function. The binary mixture moves in vertical layer between rigid walls. We assume no-slip conditions and the absence of the matter flow on the solid walls are valid. The temperature on the walls is specified. The equations (1) are reduced to a system of ordinary differential equations with boundary conditions. Some general properties of solution of this system are established, an existence theorem is proved.

Example 2 The other invariant solution of system (1) we find with respect to the generators

$$\partial_t,\ \partial_{x^2},\ \partial_{x^3} + (\varphi x^3 \partial p + A \partial_C), \tag{4}$$

where $\varphi,\ A$ are the constant gradients of pressure and concentration in x^3-direction. On the lateral walls we impose no-slip condition, the absence of diffusive fluxes, specify the temperature distribution. The presence of rigid walls at the top and bottom implies that the flow rate through every cross section of constant x^3 must be zero. There is no vertical flux of substance through any cross section at the steady state. With using stream function and generators (4) equations (1) are reduced to forth-order system of ordinary differential equations with homogenous boundary conditions. Solution of this problem depends on the sign of the longitudinal gradient of the concentration. The problem is solved for arbitrary function $F(T, C) = C - \frac{D_T}{\chi - D}T + h(\frac{D_T}{\chi - D}T)$ which presents buoyancy force. Here h is the arbitrary smooth function. There are velocity and density profiles when buoyancy force depends quadratically on temperature in the picture. We observe convective flow which consists of three parts with varying intensity. Fluid rises up by the center of layer and falls by the walls. So nonlinear density distribution significantly affects the velocity distribution of the mixture.

CONCLUSION

Recently the majority of investigations are connected with the increasing demand for obtaining some detailed numerical information about the motions of liquid, caused by an expulsive force that nonlinearly depends on the parameters of state. Such flows vary greatly in terms of their mechanisms, physical sizes, and the forms or arising motions. They are often encountered in nature and technical devices. In this work the equations describing convection in binary mixture with Soret effect are considered. The symmetry classification of this system with respect to the arbitrary function defining buoyancy force is made. These results are useful for analytical and numerical modelling of convection in binary mixture with Soret effect. Unfortunately the numerical solution that is based on fixation of the majority of the systems parameters is not always capable of performing forecasting. This leads to even stronger necessity of obtaining precise and approximate analytic solutions which can reveal the dominant role of some parameters of the systems.

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References

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