# VELOCITY SPACE-TIME CORRELATIONS IN TURBULENT THERMAL CONVECTION

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Summary We report an experimental investigation of the longitudinal space-time cross-correlation function of the velocity field, $C(r,\tau)$, in a cylindrical turbulent Rayleigh-Bénard convection cell using the particle image velocimetry (PIV) technique. We show that the isocorrelation contours of the measured $C(r,\tau)$ have a shape of elliptical curves and hence $C(r,\tau)$ can be related to $C(r_E,0)$ via $r_E^2 = (r-U\tau)^2 + V^2\tau^2$ with $U$ and $V$ being two characteristic velocities. We further show that the fitted $U$ is proportional to the mean velocity of the flow, but the values of $V$ are much larger than the theoretical predictions.

## INTRODUCTION

Turbulent kinetic energy cascades are of prime importance and have been the central issue in the study of fluid turbulence. An important quantity to describe such cascade processes is the longitudinal velocity space-time correlation function, i.e.

$$C(r,\tau) = \langle v(x+r,t+\tau)v(x,t)\rangle / v_{rms}(x)v_{rms}(x+r), \tag{1}$$

where $v_{rms}(x)$ is the root-mean-square (r.m.s.) velocity. From experimental aspects, velocity measurements are usually carried out at a single fixed location, based on which time series of fluctuating velocities are obtained and the velocity temporal auto-correlation function $C(0,\tau)$ are calculated. To relate the properties of the experimentally measured $C(0,\tau)$ in time domain to theoretical predictions of $C(r,0)$ in space domain, one needs to invoke the Taylor's frozen-flow hypothesis. The validity of such hypothesis demands low turbulent intensity and weak shear rates. However, these conditions are not always met by actual flows of interest [1]. Moreover, when Taylor's hypothesis holds, we have $C(r,\tau) = C(r_T,0)$ with $r_T = r - U_0\tau$, where $U_0$ is the mean velocity of the flow. Such a relation implies that $C(r,\tau)$ would keep constant at increasing $r$ and $\tau$ once the value of $r - U_0\tau$ remains constant, which violates the basic properties of the correlations that $C(r,\tau)$ decays to zero at sufficient large separations. Therefore, Taylor hypothesis holds only for the limited ranges of $r$ and $\tau$. Recently, based on a second order approximation to $C(r,\tau)$, He & coworkers [2] advanced an elliptic model for turbulent shear flows and proposed that $C(r,\tau)$ could be related to $C(r_E,0)$ via

$$r_E^2 = (r-U\tau)^2 + V^2\tau^2, \tag{2}$$

where $U$ is a characteristic convection velocity proportional to the mean velocity $U_0$ and $V$ is a characteristic velocity associated with the r.m.s. velocity and the shear-induced velocity. The He's elliptic model provides a useful tool for a large set of flow systems where the Taylor's hypothesis does not hold and hence the validity of the model in various practical flows needs to be tested.

In this paper, we want to validate the elliptic model in turbulent convection, an important class of turbulent flows that play an central role in many natural and engineering processes. The flow at hand is turbulent Rayleigh-Bénard (RB) convection, i.e. the convective motion of an enclosed fluid layer heated from below and cooled from above, which has received tremendous attention during the past few decades [1]. Although it has long been recognized that the conditions for Taylor's hypothesis are often not met in the system, single-point or time-domain measurements are employed by most experimental investigators for the studies of turbulent cascade processes of both the velocity and temperature fields (see Ref.[1] the recent review paper and references therein). To translate correctly the quantities measured in time domain to those in space domain, it is thus essential to validate the elliptic model in turbulent RB convection.

Recently, Tong & coworkers [3] verified indirectly the elliptic model via the local temperature data. However, a passive additive may display characteristics so different from those of the advecting velocity field [4]. In addition, the temperature field is nearly homogeneous in the cell's central region. The lack of temperature contrast would make the local temperature not follow the behaviors of the velocity field, e.g. the local velocity fluctuations show strong oscillations at the cell center [5], while the oscillation is absent for temperature measured at the same location [6]. Therefore, it is highly desirable to verify the elliptic model *directly* via the velocity field, which is the object of the present experimental investigation.

## EXPERIMENTAL SETUP AND PARAMETERS

The experiment was conducted in a water-filled cylindrical convection cell with top and bottom copper plates and plexiglas sidewall, with its height and inner diameter both being 50 cm. The measurements were carried out at fixed Prandtl number $Pr = 5.5$ and covered the Rayleigh number range $5.9\times10^9 \le Ra \le 1.1\times10^{11}$. The velocity field was measured by the particle image velocimetry (PIV) technique with a measuring area of $49\times49$ cm$^2$ and a spatial resolution 7.76 mm, corresponding to $63\times63$ measured velocity vectors. 50-$\mu$m-diameter polyamide spheres (density 1.03 g cm$^{-3}$) were used as seed particles. We chose the vertical rotation plane of the large-scale circulation as the laser-illuminated plane, defined as the $(x,z)$ plane. The Cartesian coordinate is defined such that the origin $(0,0)$ coincides with the cell center,

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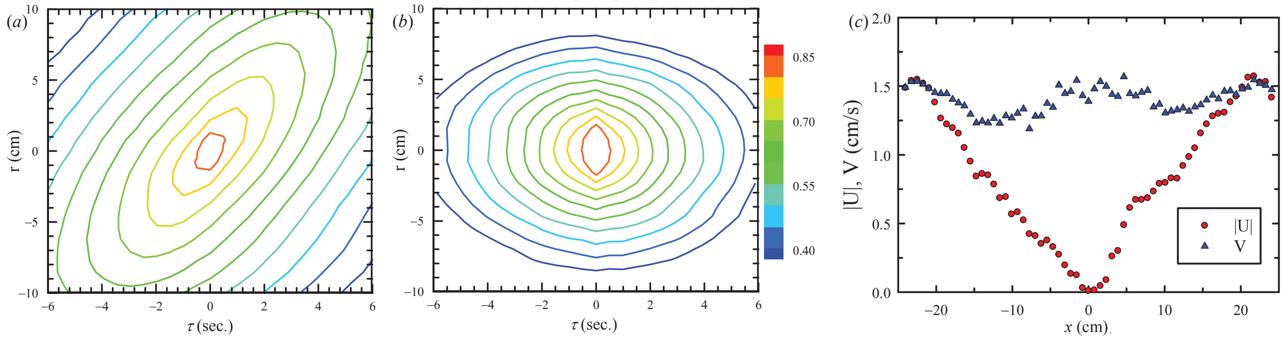


**Figure 1.** The isocorrelation contours of $C_w(r,\tau)$ measured near the cell sidewall ($a$) and at the cell center ($b$) obtained at $Ra = 9.5\times 10^{10}$.. ($c$) Comparison of $|U|$ (circles) and $V$ (triangles) for $C_w(r,\tau)$ along the cell's diameter at the middle height of the cell.

the $x$ axis points to the right, and the $z$ axis points upwards. The horizontal velocity component $u(x,z)$ and the vertical one $w(x,z)$ were obtained and the longitudinal space-time cross-correlation functions for both the horizontal and vertical velocities, $C_u(r,\tau;z)$ and $C_w(r,\tau;x)$, were calculated respectively. The experiment lasted 145 minutes in which a total of 20000 two-dimensional vector maps were acquired with a sampling rate of $\sim$ 2.3 Hz. [7]

## RESULTS AND DISCUSSIONS

We first study the properties of longitudinal space-time correlations for the vertical velocity, $C_w(r,\tau;x)$, at the middle height of the cell ($z = 0$ cm). Figure 1($a$) and ($b$) show the isocorrelation contours of $C_w(r,\tau;x)$ measured, respectively, near the cell sidewall ($x = -21.73$ cm), where the mean and the r.m.s. velocities are of the same order, and at the cell center ($x = 0$ cm), where the mean horizontal and vertical velocities are both nearly zero. One sees that the measured $C_w(r,\tau;x)$ is a single-peak function with the peak locating at the origin and decays with increasing separations $r$ or $\tau$. For the $r$- and $\tau$-range studied, the isocorrelation contours of $C_w(r,\tau;x)$ are the elongated and closed curves, rather than straight lines as suggested by the Taylor's hypothesis, and seem to have a shape of elliptical curves that can be described well by (2). Furthermore, all isocorrelation contours appear to be self-similar, i.e., they share the same preferred orientation and the same aspect ratio. One also sees that $C_w(r,\tau;x)$ decays relatively slowly in the preference direction, but drops much faster in the direction that is perpendicular to the preference direction. The slope of the preferred orientations is significant near the cell sidewall [figure 1($a$)], but vanishes at the cell center figure 1($b$). This is because the preferred orientations of the contours are directly related to the mean velocity of the flow [2].

The best fittings of (2) to the elliptical contours can yield the values of the two characteristic velocities $U$ and $V$. It is found that the fitted $U$ is proportional to the mean velocity of the flow, but the values of $V$ are much larger than the theoretical predictions. Direct comparison between the magnitudes of $U$ (circles) and $V$ (triangles) is made in figure 1($c$). It is seen that $|U| \simeq 0$ at the cell center and increases from the cell center to the sidewall and the maximization of $|U|$ occurs near the cell sidewall, while the variation of $V$ is much weaker. Note that the validity of Taylor's frozen-flow hypothesis requires $|U| \gg V$. However, the figure shows clearly that $|U| \gg V$ is not the case. This further conforms that Taylor hypothesis does not hold in the present system. The same results can be obtained for the correlations of the horizontal velocity, $C_u(r,\tau;z)$, and hence will not be shown here.

An important implication of the present work is that when $r = 0$, the relation (2) becomes $r_E = -(U^2+V^2)^{1/2}\tau$. In this case, $r$ is still proportional to $\tau$ but just that the proportionality constant is $(U^2+V^2)^{1/2}$, rather than the mean velocity $U_0$ as stated in the Taylor's hypothesis. This implies that if one is only interested in the scaling exponents of $C(r,0)$ in space domain, the Taylor's hypothesis and elliptic model would yield the same results. We note that Taylor's hypothesis has been widely used to study the scaling behaviors of structure functions and power spectrum of the velocity and temperature fields in turbulent RB convection [1]. However, it has long been known in the field that the condition for Taylor's hypothesis is not often met [1], and hence the results based on Taylor's hypothesis is questionable and not convincible. Here, our results imply that one does not really need the validity of Taylor's hypothesis to reconstruct the space series from the measured time series.

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