INFLUENCES OF MODEST ROTATION ON THE LARGE-SCALE CIRCULATION IN TURBULENT RAYLEIGH-BÉNARD CONVECTION

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Summary We report experimental studies on the properties of the large-scale circulation (LSC) for Rayleigh-Bénard convection in the present of rotation in a cylindrical sample with aspect ratio 1.0. At modest rotating speeds with the inverse Rossby number 1/Ro≤0.33 rotation enhances the strength of the LSC, as reflected by the increasing of the azimuthal temperature amplitude of the LSC. The Coriolis force gives rise to the retrograde rotation of the LSC circulation plane θ in the rotating sample frame. The retrogradation speed $\dot{\theta}$ is an order of magnitude smaller than the rotating speed Ω. We attribute the slow retrogradation speed to the viscous drag generated from the boundary layers that is further influenced by turbulent viscosity in the interior fluid.

Introduction

In rotating turbulent Rayleigh-Bénard convection (RBC), the nature of the main flow structure, the large-scale circulation (LSC) is determined by the Rayleigh number $Ra = \alpha g \Delta T L^3/\kappa\nu$, the Prandtl number $Pr = \nu/\kappa$, the Rossby number $Ro = \sqrt{\alpha g \Delta T/L}/2\Omega$, and the aspect ratio of the sample cell. Here α is the thermal expansion coefficient, g is the gravitational acceleration, κ and ν the thermal diffusivity and kinematic viscosity, respectively, ΔT is the temperature difference between the top and the bottom of the sample, Ω is the rotating velocity, and L is the height of the sample.

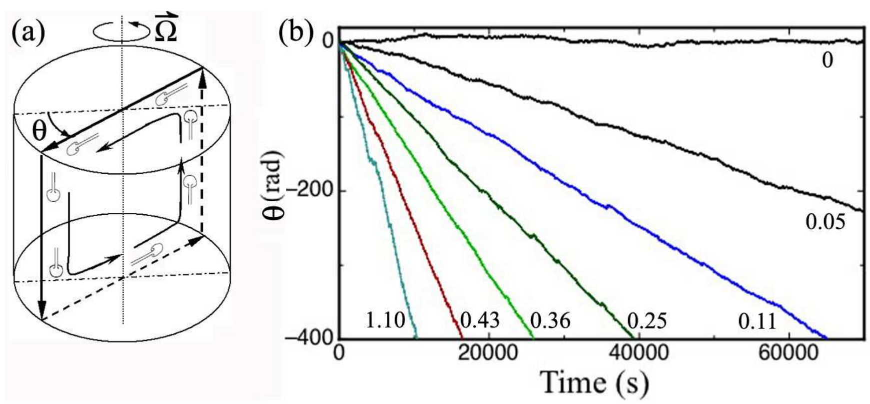

Figure 1. (a) Sketch of the main flow structure in rotating turbulent RBC that depicts the LSC orientation θ. (b) The LSC orientation θ as a function of time. The numbers in the figure indicate the values of $1/Ro$. Results are for $Ra = 1.8\times10^{10}$ and $Pr = 4.38$.

Retrograde rotation of the large-scale circulation

Figure 1a depicts the LSC flow structure in a cylindrical sample cell with an aspect ratio 1.0. Experimentally, we determine the orientation of the LSC plane θ by fitting the azimuthal temperature profile T_i $(i = 0, ..., 7)$, which is measured at eight points on the sidewall at the mid-height level of the cell, with a sinusoidal function:

$$T_f = T_0 + \delta \cos(\frac{i\pi}{4} - \theta), (i = 0, ..., 7) \quad (1)$$

The LSC temperature amplitude δ and the mean fluid temperature T_0 are both given by the fit in Eq. 1. Some measurements of θ as a function of time are shown in Fig. 1b. Rotation of the LSC plane is in the *opposite* direction of $\vec{\Omega}$ in the rotating frame. The retrograde rotation velocity $\dot{\theta}$ remains constant at each Ro.

Retrograde rotation velocity $\dot{\theta}$ as a function of $1/Ro$ is depicted in Fig. 2a for two different Ra. Mathematical models [1] predict linear dependence of $\dot{\theta}$ on $1/Ro$ as shown by the dashed lines. These models consider that a steady retrograde rotation state is achieved when the Coriolis force, which drives the LSC retrograde rotation, is balanced by the viscous drag generated from the boundary layers. The viscous drag, however, is based on the ordinary molecular viscosity of the fluid, and these models give values of $\dot{\theta}$ that are in average one order in magnitude larger than the experimental data.

The unexpected large viscous friction found from the experiment may be in concern with the *turbulent viscosity* of the fluid. We estimate the magnitude of the turbulent stress and the ordinary velocity stress to find the ratio $\langle u'u'\rangle/\langle\nu\partial U/\partial x\rangle \geq 5$, given by the LSC mean flow speed $U = 1.0$cm/s (at $Ra = 9.0\times10^9, Pr = 4.38$), the velocity fluctuation $u' = Re^{-1/4}U$ [2], and $\partial U/\partial x \leq U/\lambda$ where λ is the thickness of the boundary layer $\lambda = 0.5LRe^{-1/2}$.

Although introduction of the turbulent viscosity provides a reasonable constrain for the magnitude of $\dot{\theta}/\Omega$, the dependence of $\dot{\theta}/\Omega$ as a function of $1/Ro$ exhibits sophisticated behavior as shown in Fig. 2b. We consider that the LSC flow is influenced by rotation under different length scales: the flow velocity divided by the Coriolis parameter $U/2\Omega$, the

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boundary layer thickness λ, and the span width of the LSC away from the walls. Increasing $1/Ro$ gives rise to the LSC retrograde rotations in several regimes in which fluid momentum interactions are characterized by different length scales.

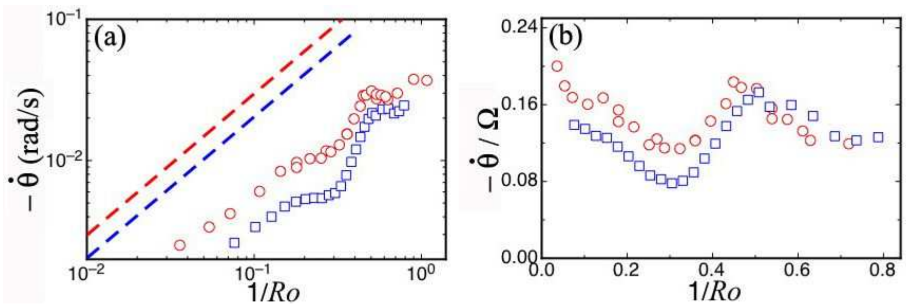

Figure 2. (a) Retrograde rotation velocity $-\dot{\theta}$ as function of $1/Ro$. (b) The ratio $-\dot{\theta}/\Omega$ as function of $1/Ro$. For both panels red circles are data for $Ra = 1.8{\times}10^{10}$, $Pr = 4.38$, blue squares are data for $Ra = 9.0{\times}10^{9}$, $Pr = 4.38$. The red and blue dashed lines in (a) are the predictions of $-\dot{\theta}(1/Ro)$ by the models in [1], for $Ra = 1.8{\times}10^{10}$ and $9.0{\times}10^{9}$ respectively.

Strength of the large-scale circulation under modest rotations

The strength of the LSC can be measured by the amplitude δ. The dependence of the normalized amplitude $\langle\delta\rangle/\Delta T$ as a function of $1/Ro$ is shown in Fig. 3a. At modest rotating rates $1/Ro < 1/Ro^* = 0.33$, results from measurements at two different Ra numbers elucidate that δ increases when the rotating speed is increased: the strength of the LSC is *enhanced* by modest rotations. Enhancement of the normalized amplitude is larger at the smaller Ra. As $1/Ro$ increases, δ reaches the maximum at a Ra-independent value $1/Ro^*$, and then starts to decreases.

The time-average fluctuation from the sidewall-temperature fit of Eq.1, $\langle\delta T\rangle = \langle(\sum_{i=0}^{7}(T_i - T_f)^2/8)^{1/2}\rangle_t$ is also depicted in Fig. 3a. Below $1/Ro^*$, the deviation of the sidewall-temperature from a harmonic sinusoidal distribution decreases when $1/Ro$ increases, indicating the LSC becomes more robust and dominant in a single-roll structure. This implies that other secondary flows, such as the corner-flow rolls in the sample cell may become suppressed under modest rotations.

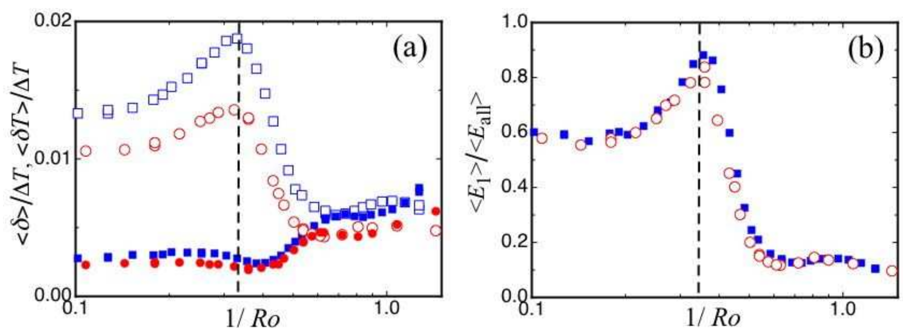

Figure 3. (a) The time-average LSC amplitude (open symbols) and the time-average rms fluctuation about the fit Eq.1 (solid symbols) as functions of $1/Ro$. (b) The normalized time-averaged energy content $\langle E_1\rangle/\langle E_{all}\rangle$. In both panels the dashed line indicates $1/Ro^*$=0.33, when δ is maximum. Red circles: data for $Ra = 1.8{\times}10^{10}$, $Pr = 4.38$. Blue squares: $Ra = 9.0{\times}10^{9}$, $Pr = 4.38$.

To further illustrate that the LSC becomes more favorable in a single-roll mode under modest rotations, we calculate the Fourier transform of the azimuthal sidewall-temperature profile and determine the average energy contents in the four lowest modes: $E_j(t) = A_j^2(t) + B_j^2(t), (j = 1, ...4)$, and $E_{all} = \sum_{j=1}^{4} E_j$. ($A_j$ and B_j are the Fourier coefficients.) The normalized time-average energy of the first mode $\langle E_1\rangle/\langle E_{all}\rangle$ is depicted in Fig. 3(b). Remarkably results from two different Ra overlap into a single curve and the energy ratio reaches a maximum at $1/Ro^*$. When $1/Ro > 0.8$, contribution to the total energy content from the first Fourier mode becomes insignificant. We suggest that the LSC lose the dominance as a single-roll structure in this rotation regime.

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