# BOUNDARY LAYER STRUCTURE IN TURBULENT RAYLEIGH-BÉNARD CONVECTION

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*Summary* We report results of studies of the boundary layer structure in turbulent Rayleigh-Bénard convection. The geometry is a closed cylindrical cell. The results are based on three-dimensional direct numerical simulations (DNS) of the Boussinesq equations at Rayleigh numbers $Ra = 3 \cdot 10^9$, $Ra = 3 \cdot 10^{10}$, at an aspect ratio $\Gamma = 1$ and at fixed Prandtl number $Pr = 0.7$. Similar to experiments in the Barrel of Ilmenau, the velocity and temperature profiles are analysed at different positions in the cell in the vicinity of the cooling and heating plates. The profiles differ from the classical Prandtl-Blasius-Pohlhausen theory, even when a so-called dynamic rescaling is performed. We show that in the present flow none of the assumptions made to derive the Blasius solution is satisfied. Furthermore, fluctuations of the local transport currents are analysed.

## SIMULATION MODEL

Resolving the dynamics inside the tiny boundary layers of thermal convection is essential for deeper understanding of turbulent heat transport. Here we study the structure of the boundary layer in turbulent Rayleigh-Bénard convection by three-dimensional direct numerical simulation (DNS) and using a cylindrical cell geometry with a heating plate at the bottom and cooling plate at the top. Our simulation parameters are Rayleigh numbers $Ra = 3 \cdot 10^9$ and $Ra = 3 \cdot 10^{10}$, with an aspect ratio $\Gamma = 1$ and a Prandtl number $Pr = 0.7$. The Boussinesq equations are made dimensionless by means of the characteristic velocity $U_f = \sqrt{g\alpha\Delta T H}$ and the cell height $H$. Here, $g$ is the gravitational acceleration, $\alpha$ the thermal expansion coefficient, $\Delta T$ the temperature difference between the heating and cooling plates. Owing to the cylindrical geometry we switch from Cartesian to cylindrical coordinates, $(x, y, z) \to (r, \varphi, z)$. Boundary conditions are the no-slip condition for the velocity components at all cell walls, isothermal top and bottom plates, and adiabatic side walls for the temperature. More details are found in Ref. [1]. The grid sizes are $N_r \times N_\varphi \times N_z = 301 \times 513 \times 360$ for $Ra = 3 \cdot 10^9$ and $513 \times 1153 \times 861$ for $Ra = 3 \cdot 10^{10}$ [2].

## MEAN BOUNDARY LAYER PROFILES

First, vertical mean profiles are compared with the experiments in the Barrel of Ilmenau. Therefore, we take time series of the turbulent fields at one-dimensional arrays of probes. For both Rayleigh number runs, there are four probe arrays starting from $z = H$ into the bulk. Probe array 1 is located at the cell centerline. Probe arrays 2, 3 and 4 are seeded at $r = 0.88R$ and $\varphi = 0, \pi/2$ and $\pi$, respectively. More detailed, time series of

$$v(r, \varphi, z, t) = \sqrt{u_r^2(r, \varphi, z, t) + u_\varphi^2(r, \varphi, z, t)} \tag{1}$$

and

$$\Xi^t(r, \varphi, z, t) = \frac{T(r, \varphi, z, t) - T(z = H)}{T(z = H/2) - T(z = H)} \tag{2}$$

are taken. They are compared with the Prandtl-Blasius-Pohlhausen profiles. It is detected that the horizontal velocity profile and the normalized temperature profile deviate from the Prandtl-Blasius-Pohlhausen predictions as seen in Figure 1. This result holds for both Rayleigh numbers.

Second, we followed the idea of Zhou & Xia [3] for their convection experiment in a narrow rectangular cell and applied the so-called dynamic rescaling of the mean velocity and temperature profiles in a plane that is aligned with the instantaneous large-scale circulation (LSC). A time-dependent boundary layer thickness is obtained by the intersection of a horizontal line through the maximum thickness value with a linear slope fit at the wall. In contrast to the quasi-two-dimensional experimental case, the profiles obtained with a dynamic rescaling in our DNS still do not agree with the Prandtl-Blasius-Pohlhausen profiles.

## BOUNDARY LAYER DYNAMICS

In order to understand the persistent deviations from the Prandtl-Blasius case better, we investigated the large-scale circulation, the pressure fluctuations in the boundary layer and the local dynamics inside the boundary layer in our DNS. The large-scale circulation in the cell is varying in its direction and amplitude significantly providing a time-dependent driving of the boundary layer dynamics. The fluctuating LSC is also in line with a strongly fluctuating thickness of both boundary layers.

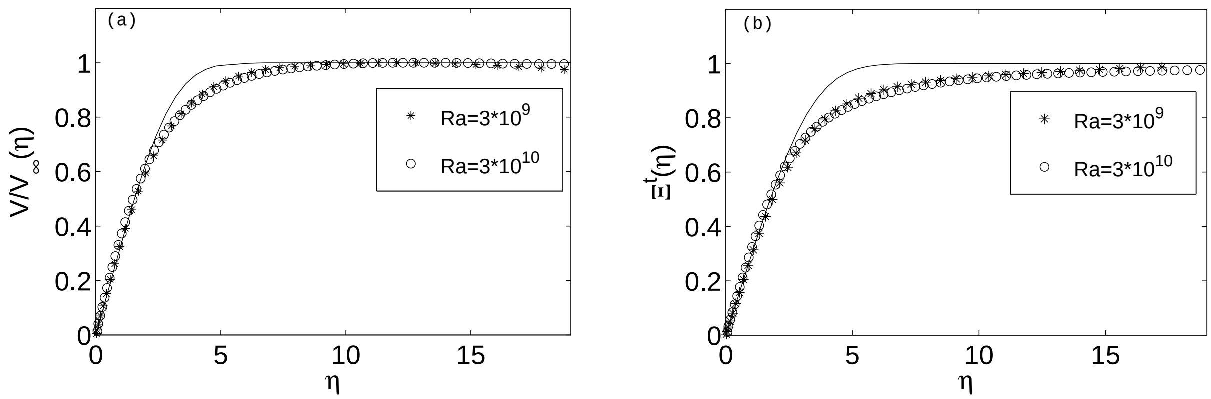


**Figure 1.** Mean profile of velocity (a) and temperature (b) at the centerline for two different Rayleigh numbers. The solid lines in both panels correspond with the classical Prandtl-Blasius (a) and Pohlhausen solutions (b).

The horizontal components of the pressure gradient are found to be non-zero. Their distribution is highly intermittent and the intermittency of the fluctuations of the pressure gradient components increases close to the heating and cooling plates. The analysis of the dynamics in the localized observation planes in the centre of the heating plate indicates that the dynamics is always three-dimensional and thus cannot be fully described by existing boundary layer theories, neither the one of Prandtl and Blasius for forced convection nor the one for natural convection. Both assume a two-dimensional flow. Since ingredients from both limiting cases are present in the boundary layer of turbulent convection – the active role of the temperature as in natural convection and the large-scale outer flow as in forced convection – the dynamics at hand is rather a mixed convection problem.

In the next step of our investigations we will study the statistics of the local transport currents in different regions of the cell and relate these currents to the dominance characteristic structures of Rayleigh-Bénard convection, the thermal plumes. This will quantify the statistical impact of the plumes for the heat transport in the boundary layer.

[1]We wish to thank the Deutsche Forschungsgemeinschaft within the Forschergruppe FOR1182 and the German-Israeli-Foundation for support.

## References

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