ELEMENTARY WAVES OF RIEMANN PROBLEM AND INTERACTION WITH A WEAK DISCONTINUITY

R.Radha[*a)], & V.D.Sharma[**]

[*] *Department of Mathematics and Statistics, University of Hyderabad, Hyderabad, 500046, India*
[**] *Department of Mathematics, Indian Institute of Technology Bombay, Powai, Mumbai, 400076, India*

Abstract

The behavior of weak discontinuity waves for the one-dimensional Euler equations governing the flow of ideal polytropic gases is analyzed. Interaction of a weak discontinuity wave with the elementary waves of the Riemann problem is considered. The effects of initial states and the shock strength on the jumps in shock acceleration and the reflected and transmitted waves are investigated.

Formulation of the problem

Here, we consider the situation when the initial discontinuity breaks up into two shocks and a characteristic shock. Given the state on one side of the discontinuity and varying the state on the other, these intermediate states are determined as solutions to the Riemann problem. It is investigated as to how a change in the initial data influences the solution of the Riemann problem. While considering the interaction of a C^1- wave with the elementary shock solutions of the Riemann problem, it is found that if the initial C^1- wave is an expansion wave then it will always impinge on the shock after a finite time, however a compression wave can meet the shock only under certain conditions. The speeds of shocks, which are elementary wave solutions of the Riemann problem, are determined and it is investigated to how the change in the initial states can influence the shock speeds. It is found that an increase in the shock strength causes the amplitudes of the transmitted and reflected waves to increase in magnitude. Further it is noticed that if the incident wave is compressive then it causes the shock to accelerate, but if the incident wave is an expansion wave then its effect would be to decelerate the shock. For a characteristic shock, the nature of the transmitted wave remains quite similar to that of incident wave, but the nature of the reflected wave is not so. All these results, stated as above, have not been reported before to the best of our knowledge; exception being the last observation that for a weak shock, the jump in its acceleration due to an interaction with a C^1- wave can be zero only when the incident wave belongs to the same family as the shock.

The physical analogue of the Riemann problem is the shock tube problem in which a long thin cylindrical tube is partitioned by a thin diaphragm located at, say, $x = 0$ (see Fig.1). The tube is filled with a gas on both sides of the diaphragm with the initial state specified. When the diaphragm is removed at time $t = 0$, the initial discontinuity located at $x = 0$ breaks up into three waves S_k, $k = 1, 2, 3$, separating four constant states, which from left to right are $\mathbf{U}_1 = \mathbf{U}_\ell$, $\mathbf{U}_2$, $\mathbf{U}_3$ and $\mathbf{U}_4 = \mathbf{U}_r$. The simplest case of a motion, in the $x - t$ plane, involving an interaction of a weak discontinuity wave $D_k^{(*)}$ with the elementary waves S_k, of the Riemann problem can be envisaged due to the presence of a kink in the cylindrical tube located at $P(x_0, 0)$, on the left of the diaphragm; see, fig.1 with $x_0 = -1$. When the diaphragm, located at $x = 0$, is removed at $t = 0$, waves S_k, $k = 1, 2, 3$, are produced originating from $(0, 0)$, propagating up and down the tube. The kink located at P gives rise to weak discontinuities $D_k^{(1)}$, originating from P and propagating into the region $\mathbf{U} = \mathbf{U}_1$, the fastest of which, i.e., $D_3^{(1)}$, interacts with the wave structure S_k; such an interaction creates a discontinuity in the acceleration of the shock front besides producing reflected and transmitted waves.

Conclusions

Here, we consider the test data for three Riemann problems, so that the solution consists of a left facing shock S_1, a characteristic shock S_2 and a right facing shock S_3. In all the three cases, envisaged here, the ratio of specific heats is taken to be $\gamma = 1.4$, and the weak discontinuity wave $D_3^{(1)}$, determined by $dx/dt = u_1 + c_1$, which collides with the shock S_1 of the Riemann problem, originates from the point $P(-1, 0)$. We have considered the situations in which (i) when the left state is known and varying the right state, (ii) when the right state is given and varying the left state, the states $\mathbf{U}_2$ and $\mathbf{U}_3$ are determined as solutions to the Riemann problem. Subsequently, we compute speed of the discontinuities σ_1, σ_2 and σ_3 and determind jumps in shock acceleration along with the amplitudes of reflected and transmitted waves; the computed values. The incident wave $D_3^{(1)}$, after its interaction with the shock S_1, gives rise to the transmitted waves $D_3^{(2)}$ and $D_2^{(2)}$ with no reflected wave. The interaction of incident wave $D_3^{(2)}$ with the shock S_2 gives rise to the reflected wave $D_1^{(2)}$ and the transmitted wave $D_3^{(3)}$. Similarly, when the incident wave $D_3^{(3)}$ interacts with the shock S_3, it gives rise to two reflected waves $D_1^{(3)}$ and $D_2^{(3)}$ with no transmitted wave. The computed results show that for shock S_1 or S_3, if the incident wave is expansive (respectively, compressive), then the transmitted waves (in case S_1) or the reflected waves (in case S_3) are also expansive (respectively, compressive); moreover, an increase in the shock strength δ_1 or δ_3 causes the amplitudes of transmitted waves/ or reflected waves to increase in magnitude. Further, if the incident wave is compressive (respectively,

[a)] Corresponding author. E-mail: repakar@yahoo.com, radhasm@uohyd.ernet.in

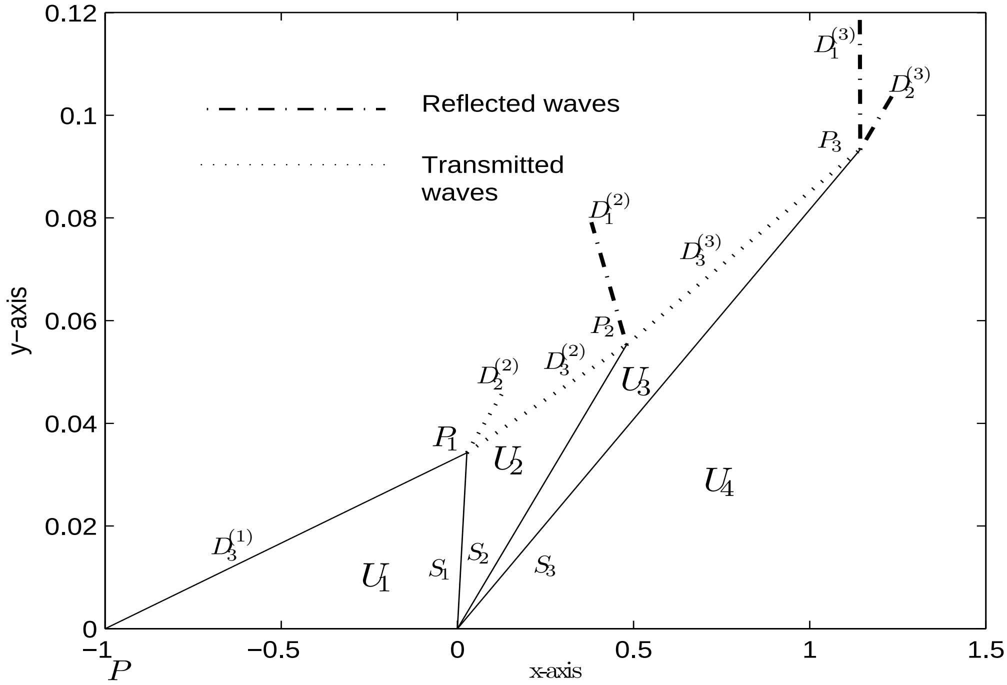

Fig.1 Shock waves S_i $(i = 1, 2, 3)$ separating constant states $\mathbf{U}_\alpha$ $(\alpha = 1, 2, 3, 4)$ involving interaction of weak discontinuities $D_3^{(i)}$ (i=1,2,3).

expansive), then its interaction with the shock S_1 or S_3 causes the shock to accelerate (respectively, decelerate), and an increase in the shock strength further reinforces the shock acceleration (respectively, deceleration). However, for the characteristic shock S_2, the nature of the transmitted wave, whether expansive or compressive, remains quite similar to that of incident wave, and an increase in the strength δ_2 results in an increase in the magnitude of wave amplitude α_{32} which, of course, remains always bounded.

References

1. P.D.Lax, Hyperbolic Conservation Laws, II *Comm. Pure. Appl. Math.* **10** (1957) 537 - 566.

2. L D Landau and E M Lifshitz, Fluid Mechanics, Pergamon Press, NewYork 1959.

3. A. Jeffrey, Quasilinear Hyperbolic Systems and Waves, Pitman, London, 1976.

4. G.Boillat and T.Ruggeri, Reflection and transmission of discontinuity waves through a shock wave. General theory including also the case of characteristic shocks, *Proc. Roy. Soc. Edinburgh* 83 A (1979) 17 - 24.

5. J.Smoller, Shock waves and reaction -diffusion equations, Springer-Verlag, New York, 1983.

6. Ch.Radha, V.D.Sharma and A.Jeffrey, Interaction of shock waves with weak discontinuities, *Applicable Analysis* 50 (1993) 145 - 166.

7. D.Serre, Systems of conservation laws, Vol-I, Cambridge University Press, UK, 1999.

8. A. Mentrelli, T. Ruggeri, M. Sugiyama, N. Zhao, Interaction between a shock and an acceleration wave in a perfect gas for increasing shock strength, *Wave motion* 45 (2008) 498 - 517.

9. V.D.Sharma, Quasi-linear hyperbolic systems, compressible flows and waves, CRC Press, 2010.